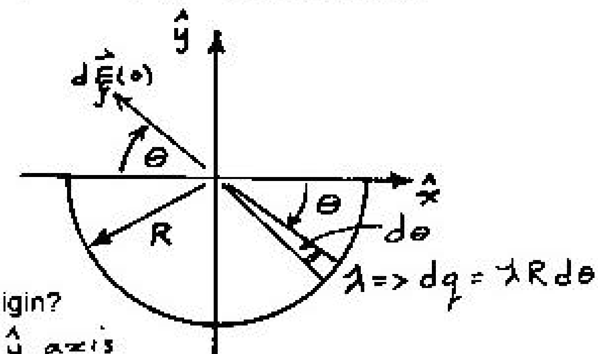
1) A thin wire on which there is a linear charge density λ describes a semicircle of radius R centered on the origin and lying in the lower half of the x-y plane as shown.



a) [15] What is the electric field at the origin?

Problem symmetric about $\hat{y}$ axis

$\Rightarrow \vec{E}(0) = E_y(0)\hat{y}$

$\Rightarrow$ consider only $\hat{y}$-components of field

$$dE_y(0) = \frac{k\,dq}{R^2} \cdot \sin\theta = \frac{k\lambda \cancel{R}}{R^{\cancel{2}}} \sin\theta\, d\theta$$

$$\Rightarrow E_y(0) = \frac{k\lambda}{R}\int_0^{\pi} \sin\theta\, d\theta = \frac{k\lambda}{R}\Big[-\cos\theta\Big]_0^{\pi} = \frac{k\lambda}{R}\left[-(-1)+1\right]$$

$$\Rightarrow \vec{E}(0) = 2k\lambda/_R\ \hat{y}$$

b) [10] A point charge is placed in the upper half of the x-y plane at a distance R from the origin such that the field at the origin is now zero. What are the magnitude and sign of the point charge and where is it located?

Since $\vec{E}(0)$ points in $\hat{y}$ direction a charge generating an equal but oppositely directed field must be located on the $\hat{y}$ axis.

$$\vec{E}_q(0) + \vec{E}(0) = 0 \Rightarrow \vec{E}_q(0) = E_q(0)(-\hat{y})$$

$$\Rightarrow -E_q(0) + E_y(0) = 0 = -\frac{\cancel{k}q}{R^{\cancel{2}}} + \frac{2\cancel{k}\lambda}{\cancel{R}}$$

$$\Rightarrow q = +2\lambda R$$

2) The National Board of Fire Underwriters has fixed safe current-carrying capacities for various sizes and types of wires. For 10-gauge (diameter = 0.10 in), rubber coated copper ($\rho = 1.72 \times 10^{-8}\ \Omega \cdot m$) wire the maximum safe current is 25 A. At this current find

a) [5] the current density,

$$A = \pi r^2 = \pi (D/2)^2 = \pi \left(\frac{0.1\ in}{2} \times 2.54 \times 10^{-2}\ m/in\right)^2$$

$$= 5.07 \times 10^{-6}\ m^2$$

$$\Rightarrow J = \frac{I}{A} = \frac{25\ A}{5.07 \times 10^{-6}\ m^2} = 4.93 \times 10^6\ A/m^2 = 493\ A/cm^2$$

b) [5] the resistance of 1000 ft of this wire,

$$R = \rho \frac{L}{A} = \frac{(1.72 \times 10^{-8}\ \Omega m)(1000')(0.3048\ m/')}{5.07 \times 10^{-6}\ m^2}$$

$$= 1.03\ \Omega$$

c) [5] the voltage difference across 1000 ft of wire,

$$V = IR = (25\ A)(1.03\ \Omega) = 25.8\ V$$

d) [5] the rate at which thermal energy will be developed in this wire,

$$P = I^2 R = (25\ A)^2 (1.03\ \Omega)$$

$$= 646\ W = 0.646\ kW$$

e) [5] the time required for 2 kW·hr of energy to be dissipated.

$$E = PT \Rightarrow T = \frac{E}{P} = \frac{2\ kW \cdot hr}{0.646\ kW}$$

$$= 3.09\ hr$$

3) Two identical small metal spheres (treat as points) initially carrying charges of $q_1<0$ and $q_2>0$ experience an attractive force of 2.0 N when 1.0 meter apart. They are brought together so that the charges get equalized ($q_1'=q_2'$) and when separated again to 1.0 meter experience a repulsive force of 2.0 N.

a) [15] What were the original charges q_1 and q_2 on the two metal spheres?

$$q_1' = q_2' = \frac{q_1+q_2}{2}$$

Initially: $\dfrac{k q_1 q_2}{R^2} = -F$; $R = 1\,m$, $F = 2.0\,N$

Finally: $\dfrac{k\left(\frac{q_1+q_2}{2}\right)^2}{R^2} = F = \dfrac{k(q_1+q_2)^2}{4R^2}$

$$q_2 = \frac{-FR^2}{k q_1} \Rightarrow F = \frac{k}{4R^2}\left(q_1 - \frac{FR^2}{k q_1}\right)^2$$

$$\Rightarrow \frac{4R^2F}{k} = q_1^2 - 2\frac{FR^2}{k} + \frac{F^2R^4}{k^2 q_1^2} \Rightarrow \frac{6R^2F}{k} = q_1^2 + \frac{F^2R^4}{k^2 q_1^2}$$

$$\Rightarrow q_1^4 - 6R^2F q_1^2/k + F^2R^4/k^2 = 0$$

$$\Rightarrow q_1^2 = \frac{1}{2}\left[6R^2F/k \pm \sqrt{36R^4F^2/k^2 - 4F^2R^4/k^2}\right]$$

$$= \frac{1}{2}\frac{R^2F}{k}\left[6 \pm \sqrt{32}\right] = \frac{R^2F}{k}\left[3 \pm 2\sqrt{2}\right]$$

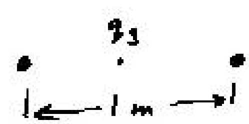
$$\Rightarrow q_1 = -\sqrt{\frac{R^2F}{k}\left(3 \pm 2\sqrt{2}\right)} = -3.6\times10^{-5}\,C,\ -6.2\times10^{-6}\,C$$

$$q_2 = 6.2\times10^{-6}\,C,\ 3.6\times10^{-5}\,C$$

b) [10] You are now required to place a third charge q_3 precisely at the midpoint between the two spheres. What is the sign and the magnitude of q_3 such that neither of the spheres experiences any force?

$$q_1' = q_2' = \frac{q_1+q_2}{2} = \pm\frac{(3.6-6.2)\times10^{-5}\,C}{2} = \pm 1.5\times10^{-5}\,C$$

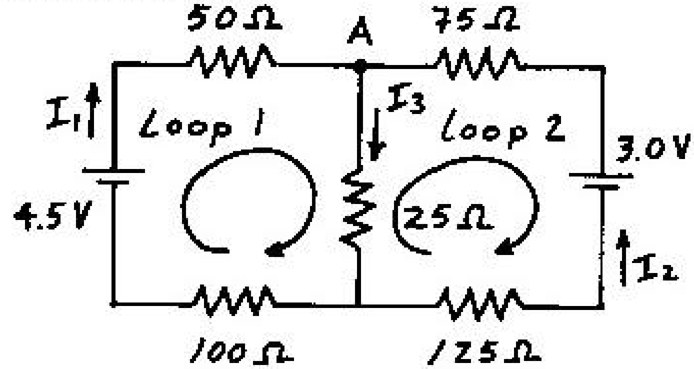
$$\Rightarrow F_{q_3} = 2\,N = \frac{k q_1' q_3}{(R/2)^2}$$



$$\Rightarrow q_3 = \left(\frac{R}{2}\right)^2(2\,N)/k q_1' = \frac{\left(\frac{1m}{2}\right)^2(2\,N)}{(9\times10^9\,Nm^2/C^2)(1.5\times10^{-5}\,C)} = 3.7\times10^{-6}\,C$$

$$\Rightarrow q_3 = \mp 3.7\times10^{-6}\,C$$

4) Consider the circuit shown:



a) [10] Give equations expressing Kirchoff's loop rules around *loop 1* and *loop 2*.

1: $0 = 4.5V - I_1 50\Omega - I_3 \cdot 25\Omega - I_1 \cdot 100\Omega$

$\Rightarrow 4.5 = 150 I_1 + 25 I_3$

2: $0 = I_3 \cdot 25\Omega + I_2 \cdot 75\Omega - 3.0V + I_2 \cdot 125\Omega$

$\Rightarrow 3.0 = 200 I_2 + 25 I_3$

b) [5] Give an equation expressing Kirchoff's junction rule at *point A*.

A: $I_3 = I_1 + I_2 \Rightarrow I_1 = I_3 - I_2$

c) [10] Find a numerical value for I_3.

1: + A: $\Rightarrow 4.5 = 150(I_3 - I_2) + 25 I_3 = 175 I_3 - 150 I_2$

$= -150 I_2 + 175 I_3 \quad \times 4$

2: $\Rightarrow 3.0 = 200 I_2 + 25 I_3 \quad \times 3$

$\Rightarrow 9.0 = 600 I_2 + 75 I_3$

$18.0 = -600 I_2 + 700 I_3$

$27.0 = 775 I_3$

$\Rightarrow I_3 = \frac{27}{775} = 0.035$ A