Prob2454: How should the charge density of a sphere of radius R vary with r to give a radial field of constant magnitude within the sphere? What happens at the origin & why?

Answer:

$$\oint \vec{E}.d\vec{s} = \frac{q^{r}}{\in_0}$$

$$E.4\pi r^2 = \frac{q^{(r)}}{\in_0}$$

$$\text{or } E = \left(\frac{q^{(r)}}{4\pi r^2 \in_0} \right) = \text{constant}$$

$$\Rightarrow \mathrm{q}^{(\mathrm{r})} = m.4\pi r^2 \quad \text{(m=constant)}$$

$$\Rightarrow \rho = \frac{q^{(r)}}{4/3\pi r^3} = \frac{3m}{r}$$

Blows up at origin because very small charge has to give a non-zero field.