Prob3431: Given a series RLC circuit with R=500 Ω, L=92 mH, C=2 μF driven by a generator with V_0=80 V and frequency 1200 Hz, find the voltages across the capacitor, resistor and the inductor at t=0.1 s after the generator is turned on at t=0.

Solution:
We start by applying the loop rule to the circuit as follows:

$V_0 Sinwt - I.R - L\frac{dI}{dt} - \frac{Q}{C} = 0$ where Q is the charge on the capacitor.

The solution to this equation may be taken as: $Q = -Q_0 Cos(wt - j)$

Where the phase angle $j = \tan^{-1}\left(\frac{X_L - X_C}{R}\right)$ can be calculated to be 51.5°.

If we can find Q_0 then we have the charge Q completely given at any time and from this we can calculate the current I=dQ/dt and hence the voltages across the individual components.

We can write $Q_0 = V_{\max}.C$ where Vmax is the maximum potential on the capacitor. This maximum is given by:

$V_{\max} = V_0\frac{X_C}{Z} = \frac{V_0}{wCZ}$ and hence $Q_0 = \frac{V_0}{Zw}$ where Z is the total series impedance

$$Z = \sqrt{(X_L - X_C)^2 + R^2}$$

Thus: $Q = -\frac{V_0}{Zw}Cos(wt - j)$ and differentiating this gives $I(t) = \left(\frac{V_0}{Z}\right)Sin(wt - j)$

We can use the current to find the voltage across the inductor by:

$V_L = L\frac{dI}{dt} = \frac{LV_0 w}{Z}Cos(wt - j)$ = 43.0 V

The voltage across the resistor is of course:

$V_R = I.R = \frac{V_0 R}{Z}Sin(wt - j)$ = -39.0 V

The voltage across the capacitor is:

$V_C = \frac{Q}{C} = -\frac{V_0}{ZCw}Cos(wt - j)$ = -4.0 V

Note that $V_L + V_R + V_C$ = 0 since at t = 0.1 s one completes 120 cycles and the circuit is back to where it started at t=0 i.e. the generator voltage was zero.