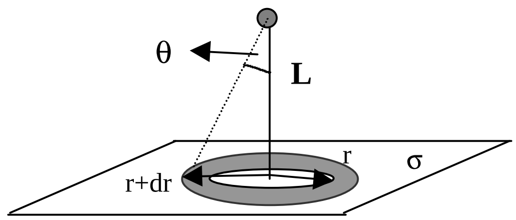
SheetCharge: Calculate the force exerted on a charge q placed a distance L away from an infinite plane sheet with surface charge density, σ.



Solution: Break up the sheet into concentric rings with charge Q and use the result that the force on the charge q by a charged ring of radius R with charge Q acts only along the y – direction and is:

$$dF_x = \frac{kqQ}{(R^2+L^2)} Cosq = \frac{kqQ}{(R^2+L^2)} \frac{L}{\sqrt{(R2+L2)}} = \frac{kqQL}{(R^2+L^2)^{\frac{3}{2}}}$$

replace $R^2 \to r^2$ and $Q \to 2prdrs$. Therefore the above equation becomes:

$$dF = \frac{2pqsrdr}{4pe_0} \frac{L}{(r^2+L^2)^{\frac{3}{2}}} \text{ and the integral is } F = \int \frac{2pqsrdr}{4pe_0} \frac{L}{(r^2+L^2)^{\frac{3}{2}}}$$

Apply change of variable: $(r^2+L^2) = t$. This means $2rdr = dt$

$$F = \frac{qsL}{2e_0} \int_0^{\infty} \frac{rdr}{(r^2+L^2)^{\frac{3}{2}}} \text{ and } F = \frac{qsL}{2e_0} \int_{L2}^{\infty} \frac{\frac{1}{2}dt}{(t)^{\frac{3}{2}}} = \frac{qs}{2e_0}$$