

Name _____ Date _____ Partners _____

Lab 7 - ELECTRON CHARGE-TO-MASS RATIO

OBJECTIVES

- To understand how electric and magnetic fields impact an electron beam
- To experimentally determine the electron charge-to-mass ratio

OVERVIEW

In this experiment, you will measure e/m , the ratio of the electron's charge e to its mass m . Given that it is also possible to perform a measurement of e alone (the Millikan Oil Drop Experiment), it is possible to obtain the value of the mass of the electron, a very small quantity.

If a particle carrying an electric charge q moves with a velocity \mathbf{v} in a magnetic field \mathbf{B} that is at a right angle to the direction of motion, it will experience the **Lorentz force**:

$$\mathbf{F} = q \mathbf{v} \times \mathbf{B}, \quad (1)$$

Which, because of the vector product, is always perpendicular to both the magnetic field and the direction of motion. A constant force that is always perpendicular to the direction of motion will cause a particle to move in a circle. We will use this fact to determine e/m of the electron by measuring the radius of that circle. To this end we must:

- produce a narrow beam of electrons of known energy,
- produce a uniform magnetic field,
- find a way to measure the radius r of the circular orbit of the electrons in that magnetic field, and
- find the relation between that radius and the ratio e/m .

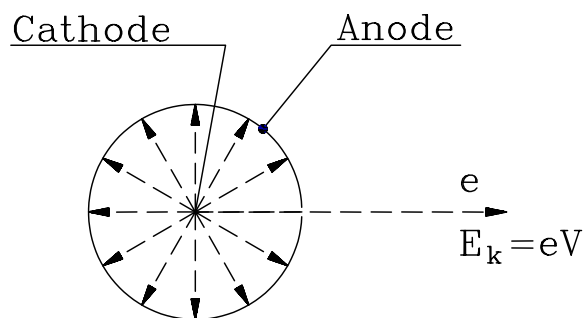


Fig. 1: Electron gun. Electrons are produced by the hot cathode at the center. A voltage V between the cathode and anode accelerates the electron through a hole in the anode, so the electrons have kinetic energy eV .

We discuss these problems in order.

The Electron Beam

When one heats a piece of metal, say a wire, to 1000 K or beyond, electrons will "boil off" from its surface. If one surrounds the wire with a positively charged electrode, an **anode**, the electrons will be attracted to it and move radially outward as indicated in Fig. 1. On their way to the anode they will acquire a kinetic energy

$$E_k = \frac{1}{2}mv^2 = eV, \quad (2)$$

where V is the potential difference, or voltage, between the heated filament, called the **cathode**, and the anode. Most of the electrons will strike the anode. However, if one cuts a narrow slit into the anode, those electrons that started out toward the slit will exit through it as a narrow beam with a kinetic energy E_k .

The Magnetic Field

According to Ampere's law a wire carrying a current I is surrounded by a magnetic field \mathbf{B} , as shown in Fig. 2. If the wire is bent into a circle the field lines from all sides reinforce each other at the center, creating an axial field (see Fig. 3). Usually one will not use a single loop of wire to create a field but a coil with many turns.

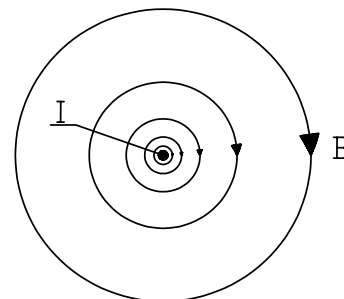


Fig. 2. Magnetic field of a straight wire.

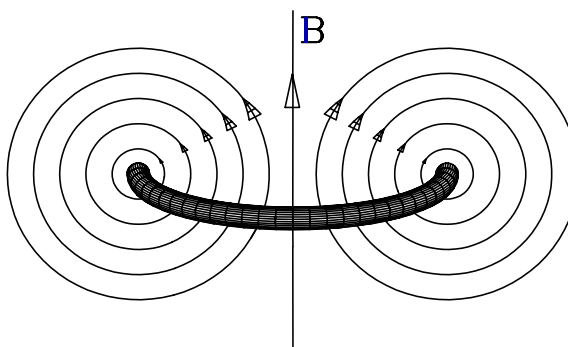


Fig. 3. Magnetic field of a wire loop.

If one uses two coaxial coils of radius R that are a distance d apart, as shown in Fig. 4, the field at the center point between the coils will be nearly homogeneous.¹ H. von Helmholtz (1821-1894) realized that there

¹ The symmetry of the arrangement makes the first derivative of the field with respect to the axial direction vanish.

remains a free parameter, namely the coil separation d , that can still be adjusted. He showed that when $d = R$, the result is a particularly homogeneous field in the central region between the coils.² Since that time **Helmholtz coils** have been used when there is a need for homogeneous magnetic fields.

One can show that the field in the center of a Helmholtz coil is given by

$$B = \frac{8NI\mu_0}{5R\sqrt{5}}, \quad (3)$$

where I is the current flowing through both coils, R is their mean radius, N is the number of turns of wire in each coil, and $\mu_0 = 4\pi \times 10^{-7} \text{ T m/A}$ is the **permeability constant**.

The Electron Orbit

Experiments like the one that you will perform have been used to measure the mass of charged particles with great precision. In these experiments the particles move in a circular arc whose beginning and end are measured very accurately in a near perfect vacuum. For our simple experiment we cannot go to such lengths and a simple expedient has been used to make the electron orbit visible. The bulb surrounding the electron source is filled with either mercury (larger apparatus) or helium (smaller apparatus) vapor. When the electrons collide with the atoms, the atoms emit light so that one can follow the path of the electron beam. These collisions will diminish the accuracy of the experiment but it remains adequate for our purposes.

A particle moving in a circle of radius r must be held there by a **centripetal force**

$$F_c = \frac{mv^2}{r}. \quad (4)$$

In our case, that centripetal force is the Lorentz force, Eq. (1), hence

$$evB = \frac{mv^2}{r} \quad (5)$$

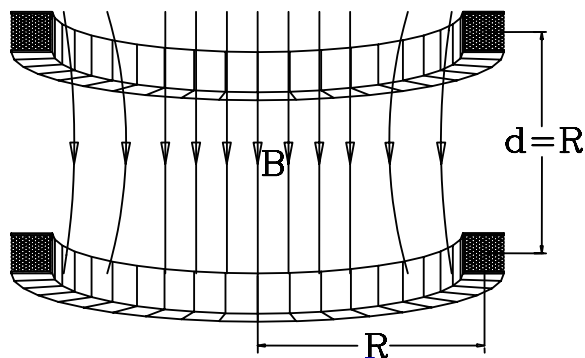


Fig. 4. Magnetic field \mathbf{B} of a pair of Helmholtz coils. R is the radius of the coils and d is the spacing between them.

² The second derivative vanishes as well.

This equation contains the velocity v , which we can eliminate by using Eq. (2). Rewriting Eq. (2), we find

$$v = \sqrt{\frac{2eV}{m}} \quad (6)$$

and hence

$$\frac{e}{m} = \frac{2V}{B^2 r^2}. \quad (7)$$

In this equation, V is the voltage between cathode and anode and r is the mean radius of the circular electron orbit, both of which can be measured, and B is the magnetic field through which the electrons pass. We know the magnetic field at the center of the Helmholtz coil, which can be obtained, using Eq. (3), from a measurement of the current through the coils, the dimension of the coils and the number of turns. The magnetic field does not change very much away from the center of the coils.

At this point the description of the experiment changes, because we do not have 16 of the same apparatus. Please refer below to the apparatus for the room number in which you have workshop.

Room 213:**INVESTIGATION 1: FINDING e/m**

For this investigation, you will need the following:

- Helmholtz coil with e/m glass tube
- Compass
- Dip needle
- Hewlett Packard E 3617A DC Power Supply (anode)
- Lambda LQ-520 Power Supply (filament)
- Heath/Schlumberger SP-2721 Power Supply (coil current)
- 2-m measuring tape

Activity 1-1: Magnetic Field Produced by the Helmholtz Coils

In this activity, you will familiarize yourself with the setup you will be using.

1. Look in the center of the Helmholtz coils for the glass tube. **The tube is extremely fragile, and the wires sticking out from the tube have current and voltage, so be very careful.** This evacuated glass tube has mercury vapor inside. The electrons will follow circular orbits inside this tube.
2. Locate the anode inside the glass tube. It is the metal cylinder with a slit in its side. At its center is the filament or cathode that will be heated by a current to emit the electrons.
3. Now locate the wire structure inside the bulb which has five vertical wire targets that will help you to determine the radius of the electron orbit.
4. There are three separate electrical circuits, one to heat the filament/cathode (Lambda LQ-520 power supply), another to apply a voltage between cathode and anode (Hewlett Packard E 3617A power supply), and a third to supply the current (Heath/Schlumberger SP-2721 power supply) for the Helmholtz coils. In case of malfunction, some of these power supplies may be substituted by other brands or models. Make sure they are all wired correctly, as shown in Fig. 5. Note that the cathode and anode circuits share a common ground.

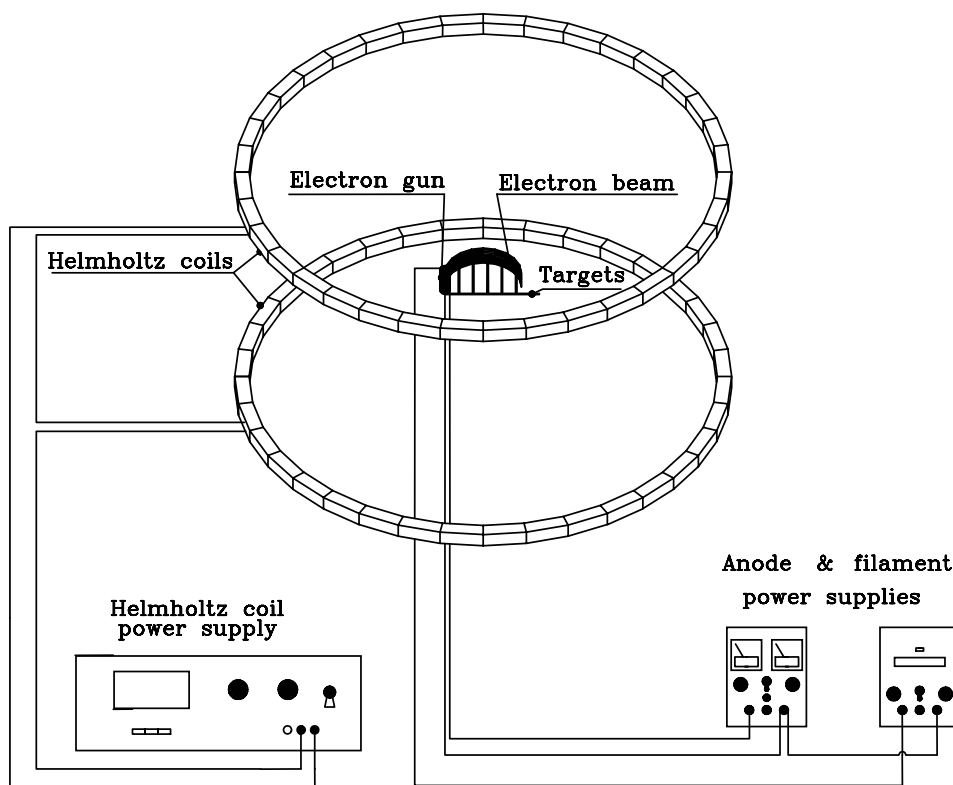


Fig. 5. Schematic view of the Apparatus.

5. Measure the mean diameter of the Helmholtz coils. Record the mean radius r below.

Radius r _____

6. Measure the mean separation d between the coils.

Coil separation d _____

Verify that $d = r$.

7. The number of turns in each coil is given on the base of the setup. Record this below.

Number (N) _____

Question 1-1: You calculated a typical magnetic field for this apparatus in your prelab. Write down its value below:

Typical Helmholtz coil magnetic field _____

The Earth's magnetic field is approximately 10^{-4} T. Discuss how much difficulty the Earth's magnetic field will cause in your experiment.

Question 1-2: In what direction should the apparatus be aligned to minimize the effects of the Earth's magnetic field? Explain.

8. One should compensate for the Earth's magnetic field. Use the compass to determine the general direction of the Earth's magnetic field. Align the base of the Helmholtz coils in the N - S direction by aligning the compass with the base of the apparatus.
9. There is also a dip needle compass that can be used to determine the direction of the Earth's magnetic field in the vertical direction. In Charlottesville the Earth's magnetic field is into the ground. Place the dip needle on the axis that contains the glass tube, so that it will rotate with that axis. One student will have to hold it in place. Another student will then adjust the tilt angle of the coils to match the dip needle's angle. When the angle of the apparatus is correct, the dip needle points to 0° and 180° . Ask your TA if you are confused about the directions.
10. To ensure that the compasses do not impact the values you find, move them away from the apparatus.
11. Make sure that all the knobs are turned down on the three power supplies. Make sure that the coil current power supply (Heath/Schlumberger) has its switch in STANDBY (this switch is on the right side of the box). If the previous students did not turn down the knobs and you did not check that they were down, you can do severe damage when you turn on the power supplies. You may now turn the power on for all three power supplies.

12. Turn up the anode voltage to 30 volts while watching the voltage reading on the power supply.
13. Now *call your instructor* to turn on the filament current. **Only your Instructor should set or change the filament voltage!**

**CAUTION! THE TUBES ARE EXPENSIVE.
EVEN A SMALL EXCESSIVE VOLTAGE CAN BURN OUT
THE FILAMENTS.**

14. Have one person in your lab group stand above the filament tube so he or she can observe what is happening. During the experiment it is essential for each group member to observe the movement of the electron beam.
15. As the filament is turned on, you should see a faint blue streak emerging from the slit in the anode due to the electron beam. Our experience is that after warming up, the filament current may need to be readjusted. If the blue streak representing the electron beam is not very bright, you may want to ask your Instructor to look at it, because it may need attention.
16. Now turn the coil current supply from STANDBY to DC ON. Increase the current through the coils while observing the electron beam curving towards the axis. Do not turn it so high that the beam radius is much smaller than when it passes the most inside vertical wire target.

Question 1-3: What happens to the beam of electrons as the coil current is increased?

Question 1-4: What causes the phenomena that you just described in Question 1-3?

17. Now adjust the coil current so the electron beam is passing near the middle (3rd) vertical target.
18. Now you may adjust the anode voltage. Do not go past 40 V.

Question 1-5: What do you observe as the anode voltage is changed while keeping the coil current is constant? Explain why this occurs.

19. While the electron beam is somewhere near the middle of your vertical targets, use the bar magnet to see how it affects the electron beam. We prefer that you not place the bar magnet above the glass tube, but you may move it around the bottom and the sides.

Question 1-6: Describe what you observe as you move the bar magnet around. Can you produce a helical path for the electron? Can you see why you want to keep spurious magnetic fields away from the electron beam?

20. Make sure you place the bar magnet at the other end of the table from your apparatus to minimize any possible effects.

Activity 1-2: Measurement Of Charge-To-Mass Ratio

1. Return the current through the Helmholtz coils to zero, and the anode voltage to 30 V. You will probably see that the electron beam has a slight curvature due to the Earth's magnetic field. You will now want to adjust the current in the coils so that the electron beam is exactly straight. To do this, note that the filament also emits visible white light. If you put your hand or a piece of paper next to the glass tube you will be able to clearly see this white light. It will be about a cm wide on the glass tube. The "straight electron beam" is established if the electron beam hits very close to the middle of this white light on the inside of the glass tube. This is a little hard to describe, so please ask your Instructor if you are unsure. A straight beam means that the magnetic field is zero, i.e. that the Earth's field is canceled out by magnetic field from the coils. Turn up the coil current until the electron beam hits close to the middle of the white light on the inside of the glass tube. We will call this coil current I_0 . You should subtract

I_0 from your subsequent measurement in order to obtain the net magnetic field.

I_0 _____

2. We will make measurements for the electrons moving in various circular orbits. We know the distances of the vertical wire targets from the filament, and they are given in Table 1-1. You will need these in order to determine e/m from Eq. (7). Note that these distances are the diameters of the circular orbits, and you need the radius r in Eq. (7).

Table 1-1

Target Number	Distance to Filament (Diameter)
1	0.065 m
2	0.078 m
3	0.090 m
4	0.103 m
5	0.115 m

HINT: The electrons collide with the mercury atoms introduced to make the beam visible. Unavoidably, the electrons lose some energy in the collisions (that make the beam visible). In order to minimize this effect, you should concentrate on those electrons that have the highest energy: those at the outer edge of the beam. **Find those values of the Helmholtz coil currents that will make the outer edge of the beam hit the far outside edge of each of the wires on the target ladder.** Ask your Instructor if you are uncertain about this.

3. Set the anode voltage to 25 V. Record your values for the coil current in Table 1-2 for the electron beam as described in the above hint in the nominal current column. Do not yet calculate the actual net coil currents; we will do this later. Record the coil current for each of the five vertical targets. This can proceed quite rapidly by one person observing the beam from above, another person adjusting the coil current, and another person filling in Table 1-2. The distances of the **far outside edges** of the targets from the filament are those given in Table 1-1.

Table 1-2

Anode Voltage	25 V		30 V		35 V		40 V	
Target number	Nominal Current	Net Current	Nominal Current	Net Current	Nominal Current	Net Current	Nominal Current	Net Current
1								
2								
3								
4								
5								

Question 1-7: The actual net current is found by subtracting the value for I_0 from the current read (nominal current). In subsequent sections of the experiment, you will use different accelerating anode voltages. Do you think that you will have to measure a new value of I_0 for each? Please explain. Would it be a good idea to check your reasoning?

- Repeat the above measurements for anode voltages of 30, 35, and 40 V. Record your values in Table 1-2.
- Turn down the knobs for all your power supplies to zero. Switch the coil current power supply back to STANDBY. Turn off all three power supplies.
- Now set up a table in Excel for the final calculations to subtract your value(s) of I_0 and calculate the values for e/m for each of your data points. We suggest columns for target diameters, target radii, anode voltage, measured current, net current, magnetic field, and anything else you think you need to calculate e/m using Eq. (7). The last column should be the difference between your value of e/m and the accepted one (1.759×10^{11} C/kg).
- Average all your results to obtain a final value for e/m .

e/m _____

8. The accepted value of e/m is 1.759×10^{11} C/kg. Find the statistical error of your average value.

Uncertainty _____

9. Make sure to print out your data table. You only need print one per group.

Question 1-8: What could be the source of error in your experiment?

Activity 1-3: Investigation of Systematic Error

It is useful in an experiment like this to see if there are any systematic errors that might affect our final results.

1. Look at your answer to Question 1-8 above. It may be possible that we may learn something about our errors if we compare the values of e/m versus our parameters. Because we have our data in Excel, it is quite easy to do that.
2. Plot all your values of e/m versus a) anode voltage, b) net coil current, and c) orbit radius. Produce trendline fits for your data. Plot out one copy each of these graphs and include them in your report.

Question 1-9: Look carefully at the three plots you just made and at your trendlines. Do you see any patterns? Could these possibly indicate any systematic problems? Discuss these possible errors and how you might be able to correct for them or improve the experiment.

**PLEASE CLEAN UP YOUR LAB AREA! LEAVE THE WIRES
CONNECTED TO THE APPARTUS.**

**MAKE SURE THE KNOBS ON THE POWER SUPPLIES ARE
TURNED DOWN AND THE POWER SUPPLIES ARE TURNED
OFF!**

Room 215:**INVESTIGATION 1: FINDING e/m**

For this investigation, you will need the following:

- Daedalon Helmholtz coil with e/m glass tube
- Compass
- Bar magnet
- 2-m measuring tape

Activity 1-1: The Helmholtz Coil Apparatus

In this activity, you will familiarize yourself with the setup you will be using.

1. Look in the center of the Helmholtz coils for the glass tube. **The tube is extremely fragile, and the wires sticking out from the tube have current and voltage, so be very careful.** This evacuated glass tube has helium vapor inside. The electrons will follow circular orbits inside this tube.

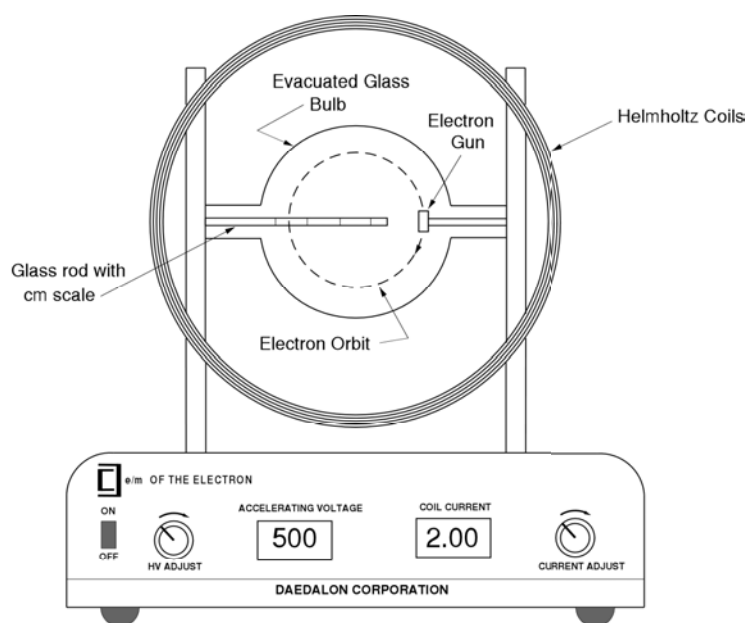


Fig. 6: e/m Apparatus

2. Locate the grid and anode inside the glass tube. It is the pair of vertically oriented metal cylinders with a gap between them. At its top and center is the filament or cathode that will be heated by a current to emit the electrons.

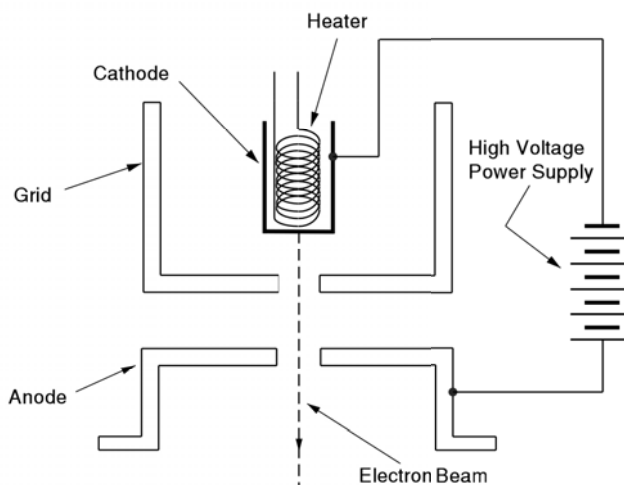


Fig. 7: Grid and Anode

- There are three separate electrical circuits: 1) to heat the filament/cathode (over which you have no control); 2) to apply a voltage between cathode and anode (denoted as Accelerating Voltage on the unit); 3) to supply the current (denoted as Coil Current) for the Helmholtz coils.

- Measure the diameter of the Helmholtz coils in several places and take the average. Record the mean radius r below.

Radius r _____

- Measure the mean separation d between the coils. You may want to average several measurements here also.

Coil separation d _____

Verify that $d = r$. (Note: We have found they may not quite agree.)

- The manufacturer states the number of turns in each coil is 130.
- You calculated a typical magnetic field for this apparatus in your prelab. A typical value for the Daedalon apparatus you will be using is 1.3 mT (milliTesla).
- Determine the relationship between the current passing through the Helmholtz coils and the magnetic field produced. You will need the above parameters to do this. You should have already done this in the prelab. Look at Eq. (3).

$$B \text{ (tesla)} = \underline{\hspace{2cm}} I \text{ (amps)}$$

9. Turn the main power on. The unit will perform a self test lasting no more than 30 s. Do not do anything with the unit during the self test. When it is finished, the coil current display will be stabilized and indicate “000”. The unit is now ready to use, but note that **there is a 10 minute warm-up time before you should take final measurements.** You can go ahead and proceed with the remainder of this procedure.
10. Turn up the voltage adjust knob to a voltage of 150 V. Look for the electron beam, which should be pointing down. Turn up the current adjust knob and look at the path of the electron beam.

Activity 1-2: The Earth's Magnetic Field

The Earth's magnetic field is approximately 10^{-4} T. We want to examine how much difficulty the Earth's magnetic field will cause in your experiment.

Prediction 1-1: In what direction should the apparatus be aligned to minimize the effects of the Earth's magnetic field? You might want to use the $\mathbf{F} = q \mathbf{v} \times \mathbf{B}$ rule to help you decide. Justify your decision.

1. One might need to compensate for the Earth's magnetic field. Use the compass to determine the general direction of the Earth's magnetic field. Align the base of the Helmholtz coils in the direction that you believe will minimize the effect of the Earth's magnetic field on your measurement.
2. We have used a magnetic dip needle compass to determine the direction of the Earth's magnetic field in the vertical direction. In Charlottesville the Earth's magnetic field is into the ground at about 58° with respect to the horizontal. It will not be possible for you to align this apparatus to account for this direction.

Question 1-1: Discuss the effect of the Earth's total magnetic field direction that is both horizontal and vertical directions, on your measurement.

3. To ensure that the compasses do not impact the values you find, move them (it) away from the apparatus.
4. You should now be able to see the filament (two wires that are orange due to being heated).

Note: Both the voltage and current outputs are controlled by a microprocessor, which locks out the controls at both the minimum and maximum settings. There is not a manual "stop" on the knobs. When the knob reaches the maximum setting, it will still turn, but the appropriate value will not change. This feature prevents excessive voltage being applied to the tube.

5. Turn the Current Adjust control up and observe the circular deflection of the beam. When the current is high enough, the beam will form a circle. The diameter of the electron path in the magnetic field can be measured using the etched glass internal scale in the tube. The graduations and numerals of the scale are illuminated by the collision of the electrons, making observation reading fairly easy. Vary the Current Adjust and note the electron beam striking several of the centimeter scale markings. You should also be able to see a vertical line indicating the half-centimeter mark. The scale numbers fluoresce when the beam hits them. Set the beam on the 11 cm mark.

NOTE: Sometimes the electron beam slightly misses the internal glass scale tube. If it is really bad, ask your TA to look at it. DO NOT TRY TO MAKE ANY ADJUSTMENTS! Sometimes it helps to put a dim backlight behind the scale to help see the numbers and half-centimeter mark.

Question 1-2: Why does the path of the electrons change as the coil current is increased?

6. With the beam hitting the 11 cm mark, put two hands on the **base** of the Helmholtz coil apparatus and, **being very careful**, slowly rotate the apparatus by 90° . Observe very closely what happens to the path of the electrons. Have one person in your lab group look down from above the glass tube so he or she can observe what is happening. During this experiment it is essential for each group member to observe the movement of the electron beam.

Question 1-3: Explain what happens to the electron beam path when you rotate the apparatus. Does this help you decide how the Helmholtz coils should be oriented with respect to the Earth's magnetic field? If so, explain.

Activity 1-3: Effects Of External Magnetic Field

1. Set the accelerating voltage to 300 V. Use the coil current to set the electron beam to a diameter of 10 cm.
2. Use the bar magnet and approach the glass tube along the axis of the Helmholtz coil. **Be careful to not touch the glass tube.** Do it with the North Pole first and then the South Pole. It is somewhat difficult to decide what is happening because the magnetic field due to the bar magnet spreads out in three dimensions.

Question 1-4: Describe what happens to the path of the electron beam for each case. In particular, note any change in the radius of the electron beam about the axis of the coil.

3. Now move the bar magnet from different directions above and below the glass tube along the direction of the plane of the Helmholtz coils (perpendicular to what you did before). Again use both ends of the magnet.

Question 1-5: Describe the path of the electron beam for each case. In particular note the radius of the electron beam about the axis of the coil.

Question 1-6: Based on these observations do you want to change your Prediction 1-1 (do not actually go back and do so)? Explain how your observations with the bar magnet confirm how you think the apparatus should be aligned with respect to the Earth's magnetic field.

4. Make sure you place the bar magnet on the other end of the table from the apparatus.
5. Set the coil current to 1.7 A. Adjust the accelerating voltage while looking at the electron beam path. Do not exceed 350 V.

Question 1-7: What do you observe as the accelerating voltage is changed while keeping the coil current constant? Explain why this occurs.

Activity 1-4: Measurement Of Charge-To-Mass Ratio

1. You want to have the Helmholtz coil placed in the best position to reduce the effect of the Earth's magnetic field based on the measurements and thinking you have done in the preceding activities.

Question 1-8: For completeness sake, describe briefly your final position of the Helmholtz coil with respect to the Earth's magnetic field. We will grade your previous explanations of why you chose this position.

HINT: The electrons collide with the helium atoms introduced to make the beam visible. Unavoidably, the electrons lose some energy in the collisions (that make the beam visible). In order to minimize this effect, you should concentrate on those electrons that have the highest energy: those at the outer edge of the beam (largest radius). Ask your Instructor if you are uncertain about this.

2. We will make measurements for the electrons moving in various circular orbits. You will write down the diameter of the path by reading the illuminated tube. You will need these in order to determine e/m from Eq (7).
3. Set the anode voltage to 150 V. Change the coil current until the beam hits the 5 cm mark. This will be the diameter, not the radius. Record the coil current and diameter in Table 1-1. Increase the coil current in turn to go through each 0.5 cm of the diameter. The 0.5 cm marks are a vertical line between the numbered cm marks. This can proceed quite rapidly by one person observing the beam while changing the coil current, and another person filling in Table 1-1.
4. Proceed in turn to observe and record data for the other accelerating voltages in Table 1-1. Sometimes the beam will physically not be present for every position listed in Table 1.1. Leave the table blank when this occurs.

Table 1-1

	Accelerating (anode) Voltage				
	150 V	200 V	250 V	300 V	350 V
Diameter (cm)	Coil Current (A)	Coil Current (A)	Coil Current (A)	Coil Current (A)	Coil Current (A)
5.0					
5.5					
6.0					
6.5					
7.0					
7.5					
8.0					
8.5					
9.0					
9.5					
10.0					

5. Turn down the current adjust knob and voltage adjust knob until the coil current and accelerating voltage displays read zero.
6. Set up a table in Excel to calculate the values for e/m for each of your data points. The columns in Excel should not look like the table above. **We suggest columns for electron path diameter and radius, accelerating voltage, coil current, magnetic field, and anything else you think you need to calculate e/m using Eq. (7). Enter the voltages one after the other in the same column.** Don't forget to change your values to the SI system before determining your final value of e/m . The last column should be the difference between your value of e/m and the accepted one (1.759×10^{11} C/kg).
7. Average all your results in Excel to obtain a final value for e/m .

e/m _____

8. Find the statistical error of your average value.

Uncertainty _____

9. Compare your value with the accepted value of e/m is 1.759×10^{11} C/kg. Percentage difference: _____%
10. Make sure to print out one data table for your group.

Question 1-9: What could be the sources of error in your experiment?

Activity 1-5: Investigation Of Systematic Error

It is useful in an experiment like this to see if there are systematic errors that might affect your final results.

1. Look at your answer to Question 1-9 above. It may be possible that we may learn something about our errors if we compare the values of e/m versus our parameters. Because we have our data in Excel, it is quite easy to do that.
2. Plot all your values of e/m versus a) accelerating voltage, b) coil current, and c) orbit radius. Produce trendline fits for your data. Plot out one copy each of these graphs and include them in your report.

Question 1-10: Look carefully at the three plots you just made and at your trendlines. Do you see any patterns? Could these possibly indicate any systematic problems? Discuss these possible errors and how you might be able to correct for them or improve the experiment. List any other systematic errors you can think of and discuss.

**MAKE SURE THE VALUES OF VOLTAGE AND CURRENT READ ZERO AND
TURN OFF THE POWER!**

