OBJECTIVES

• To understand the design of capacitive and inductive filters
• To understand resonance in circuits driven by AC signals

OVERVIEW

In the previous lab you learned about Alternating Current (AC) signals that change over time. Such signals exist all around you. Many useful devices such as radio receivers and amplifiers, computers and televisions use AC signals that are manipulated in precise ways. Resistors, capacitors, and inductors have predictable effects on AC signals and consequently are important components in electronic devices.

In this lab you will continue your investigation of the behavior of resistors, capacitors and inductors in the presence of AC signals. In Investigation 1, you will see how capacitors and inductors can act as “filters.” More precisely, you will see how these elements can be used to suppress the voltage of certain frequency ranges of AC signals, while leaving other signals relatively unchanged.

In Investigation 2, you will discover the relationship between peak current and peak voltage for a series circuit composed of a resistor, inductor, and capacitor. You will also explore the phase difference between the current and the voltage. Furthermore, you will see how such a circuit can produce interesting responses, such as filtering out all but a narrow range of signal frequencies. This circuit is often referred to as a “resonant circuit.” The phenomenon of resonance is a central concept underlying the tuning of a radio or television to a particular frequency.
**INVESTIGATION 1: INTRODUCTION TO AC FILTERS (AND STEREO SPEAKERS)**

In a previous lab, you explored the relationship between impedance (the AC equivalent of resistance) and frequency for a resistor, capacitor, and inductor. These relationships are very important to people designing electronic equipment, particularly audio equipment. You can predict many of the basic characteristics of simple audio circuits based on what you have learned in previous labs.

Recall that if there is a current of the form

\[ I(t) = I_{\text{max}} \sin(\omega t) \]

flowing through a circuit containing resistors, capacitors and/or inductors, then the voltage across the circuit will be of the form

\[ V(t) = I_{\text{max}} Z \sin(\omega t + \varphi) . \]

\( Z \) is called the *impedance* and \( \varphi \) is called the *phase shift*. The maximum voltage will be given by

\[ V_{\text{max}} = I_{\text{max}} Z . \]

When \( \varphi \) is zero, the voltage and current will be in phase. When \( \varphi \) is less than zero, the voltage will reach its peak before the current and we say that the current *lags* the voltage. When \( \varphi \) is greater than zero, the voltage will reach its peak after the current and we say that the current *leads* the voltage.

For a series combination of a resistor, a capacitor and an inductor,

\[ Z = \sqrt{R^2 + \left( X_L - X_C \right)^2} \]

and

\[ \tan(\varphi) = \frac{X_L - X_C}{R} \]

where

\[ X_C \equiv \frac{1}{\omega C} \]

and

\[ X_L \equiv \omega L . \]

\( X_C \) is called the *capacitive reactance* and \( X_L \) is called the *inductive reactance*. If there is only a capacitor or only an inductor, the impedance is simply the corresponding reactance.

The purpose of this lab is for you to create circuits that filter out AC signals with frequencies outside the range of interest. In the context of these activities, a filter is a circuit that attenuates the voltage of some range of signal frequencies, while leaving other frequency ranges relatively unaffected.
You will need the following materials:

- voltage probe
- RLC Circuit Board
- 6 alligator clip leads
- PASCO PI-9587C Digital Function Generator-Amplifier

Activity 1-1: Capacitors as Filters

In this activity, you will investigate how a circuit containing a resistor, capacitor, and signal generator responds to signals at various frequencies.

Consider the circuit in Figure 1-1 with a resistor, capacitor, signal generator and voltage probes.

\[
\begin{align*}
V_{\text{signal}} & = 5 \text{ V (amplitude)} \\
C & = 1.23 \mu\text{F} \\
R & = 33 \Omega \\
f_{\text{signal}} & = 200 \text{ Hz}
\end{align*}
\]

Figure 1-1: Capacitive filter circuit

NOTE: In most of the experiments today, we want to compare the current \( I \) through the circuit with a voltage across a circuit element, etc. Normally we would use a current probe, but its 1 Ohm resistance is not negligible with respect to \( R \) and so we will simply measure \( V \) and calculate \( I \) from Ohm’s law. Keep this in mind as you proceed with today’s experiments.

Prediction 1-1: On the axes that follow, use dashed lines to sketch your qualitative prediction for the peak current through the resistor, \( I_{\max} \), as the frequency of the signal from the signal generator is increased from zero. [Remember that \( \omega = 2\pi f \).]
Test your predictions.

1. Open the experiment file **Filter L10**.

2. We will use the internal signal generator of the computer interface, not the separate PASCO unit that we shall call the external signal generator. We will use it for a later experiment. Note that the internal signal generator parameters will appear on the computer screen.

3. The signal generator should already be set to a frequency of 200 Hz and amplitude of 5 V (+5 V maximum and -5 V minimum).

4. Before setting up the circuit, use the multimeter to measure the value of the “33 Ω” resistor, $R$, to a precision of 0.1 Ω.

   $R$ __________ Ω

5. Connect the resistor, capacitor, signal generator and probe as shown in Figure 1-1. Look at **Setup** in Data Studio for assistance.

6. Press **Start** to turn on the scope display.

7. You should see two displays on the scope display. One will be the signal generator voltage produced by the signal generator. This is the input (source) voltage for the circuit. It should be 5 V at its peak. The other voltage, due to VP1, will be the voltage across $R$ resistor and is proportional to the current through the circuit. In effect we are using the voltage measurement to measure the current through the circuit.

8. You may need to adjust the time and voltage scales on the scope display so that both the waveforms are visible. You may also need to adjust the trigger level on the left part of the screen to see the waveforms. Play with the trigger level a bit to see how it operates.

9. Use the **Smart Tool** to determine the peak (maximum) voltage through the resistor (not the signal voltage, which should remain at 5 V), write it in Table 1-1 with $f_{\text{signal}} = 200$ Hz. Then calculate the maximum current from the maximum voltage using the value of the resistor you measured in step 4.

10. Increase the frequency of the signal generator to 1200 Hz. Be sure that the peak signal amplitude is still 5 V. Repeat step 9.

11. Repeat step 9 for 2200 Hz, 4200 Hz and 8200 Hz.
Table 1-1

<table>
<thead>
<tr>
<th>$f_{\text{signal}}$ (Hz)</th>
<th>$V_{\text{max}}$ (V)</th>
<th>$I_{\text{max}}$ (A)</th>
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12. Sketch the data from Table 1-1 on the axes below. Mark scales on the vertical axes.

$\max (\text{mA})$

$\text{f}_{\text{signal}}$ (kHz)

Question 1-1: If you could continue taking data up to very high frequencies, what would happen to the peak current, $I_{\text{max}}$ through the circuit?

Question 1-2: At very high frequencies, does the capacitor act more like an open circuit (a break in the circuit’s wiring) or more like a short circuit (a connection with very little resistance)? Justify your answer.
13. Now note the phase difference (in the next question) between the peaks of the signal generator voltage and the voltage across the resistor (~circuit current) at the frequency 8200 Hz that you should still have (note that they should be close to being in phase). Then go back to a frequency of 200 Hz and observe the phase difference.

**Question 1-3:** What phase difference do you note between the peaks of the signal voltage and circuit current for low and high frequency?

**Question 1-4:** What would the current be through the circuit if we applied only a DC voltage?

**Question 1-5:** At very low frequencies, does the capacitor act more like an open circuit (a break in the circuit’s wiring) or more like a short circuit (a connection with very little resistance)? Justify your answer.

**Comment:** In the circuit in Figure 1-1, since the peak signal voltage from the signal generator remains unchanged, the peak current in the circuit must increase as the total impedance decreases. Therefore, the peak voltage across the resistor increases as the frequency of the signal increases. This type of circuit is an example of a “high-pass” circuit or filter.
Activity 1-2: Inductors as Filters

This activity is very similar to the previous one except that you will replace the capacitor with an inductor and determine the filtering properties of this new circuit.

Consider the circuit containing a resistor, inductor, signal generator and probes shown in Figure 1-2 below.

\[ V_{\text{signal}} \]

\[ L = 8.2 \text{ mH} \]
\[ R = 33 \Omega \]
\[ V_{\text{signal}} = 5 \text{ V (amplitude)} \]
\[ f_{\text{signal}} = 200 \text{ Hz} \]

Prediction 1-2: On the axes that follow, use dashed lines to sketch your qualitative prediction for the peak current through the circuit, \( I_{\text{max}} \), as the frequency of the signal from the signal generator is increased from zero.

Test your predictions.

1. You can continue to use the experiment file **Filter L10**.

2. Set the signal generator to a frequency of 20 Hz and amplitude of 5 V (+5 V maximum and -5 V minimum).

3. Connect the resistor, inductor, signal generator and probe as shown in Figure 1-2. Simply replace the capacitor in the previous setup with the inductor.

4. Press Start to turn on the scope display.
5. Adjust the time and voltage scales on the scope so that both waveforms are visible. Remember the trigger level.

6. Use the **Smart Tool** to determine the peak voltage and peak current, and enter in Table 1-2. Then calculate the maximum current from the maximum voltage using the value of the resistor.

```
<table>
<thead>
<tr>
<th>$f_{\text{signal}} ; (\text{Hz})$</th>
<th>$V_{\text{max}} ; (\text{V})$</th>
<th>$I_{\text{max}} ; (\text{A})$</th>
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7. Increase the frequency of the signal generator to 200 Hz. Make sure that the amplitude is still 5 V.

8. Repeat step 6 with 1200 Hz, 2200 Hz, 4200 Hz and 8200 Hz.

9. Sketch the data from Table 1-2 on the axes below.

```
$I_{\text{max}} \; (\text{mA})$
```

```
$f_{\text{signal}} \; (\text{kHz})$
```

**Question 1-6:** If you could continue taking data up to very high frequencies, what would happen to the peak current, $I_{\text{max}}$, through the resistor?
**Question 1-7:** At very high frequencies, does the inductor act more like an open circuit (a break in the circuit’s wiring) or more like a short circuit (a connection with very little resistance)? Justify your answer.

10. Now note the phase difference between the peaks of the signal voltage and the voltage across the resistor (~circuit current) at the frequency 8200 Hz that you should still have and then go back to a frequency of 20 Hz and observe the phase difference.

**Question 1-8:** What phase difference do you note between the peaks of the signal voltage and circuit current for low and high frequency? Note: We did this in last week’s experiment as well.

**Question 1-9:** What would the current through the circuit be if we applied only a DC voltage?

**Question 1-10:** At very low frequencies, does the inductor act more like an open circuit (a break in the circuit’s wiring) or more like a short circuit (a connection with very little resistance)? Justify your answer.
Comment: In the circuit in Figure 1-2, since the peak voltage from the signal generator remains unchanged, the peak current in the circuit must decrease as the total impedance increases. Therefore, the peak voltage across the resistor decreases as the frequency of the signal increases. This type of circuit is an example of a “low-pass” circuit or filter.

Activity 1-3: Introduction to Audio Speaker Design

Music is sound waves composed of many different frequencies (pitches) superimposed on each other all at once, and constantly changing. Low notes correspond to sound waves with a low frequency. High notes correspond to sound waves with a high frequency. A microphone is a device that can detect sound waves over a wide range of audible frequencies and convert them to AC signals. Thus, the signal from a microphone generally consists of many AC frequencies superimposed on each other all at once. (These signals can then be encoded and recorded on a storage device, such as a CD.)

Some of the ideas you have studied in this investigation are readily applied to the design of audio speakers. Most audio speakers have at least two separate “drivers” from which sound emanates. Generally, a small driver, or tweeter, generates the high frequency sound waves, and a large driver, or woofer, generates the low frequency sound waves.

Speaker designers use drivers of different size because small drivers are generally better suited for playing the high notes and, conversely, large drivers handle the low notes better. To best utilize the properties of the different sized drivers, speaker designers use capacitive and inductive filter circuits to send high frequency signals to the small driver and low frequency signals to the large driver.

In order to obtain a better understanding of how filters can be used in the design of stereo speakers, you will use the Pasco Digital Function Generator and the signal generator on the computer to make a signal which contains both a high and low frequency component. This signal will be fed into the previous filters and the output examined.

We will use the circuit shown in Figure 1-3 (below) to explore some properties of such filters. We add the outputs of two signal generators to generate a signal with both high and low frequency “components”. Note that we must introduce some resistance (~10 Ohms) between the generators and the point where we “add” the signals. We do this to keep from overloading the generators (recall that they behave like “ideal” voltage sources and will strive to maintain control of the voltage at their output terminals.)
1. Measure the values of $R$, $L$, and $C$ and record next to the nominal values in Figure 1-3 (above). Connect the circuit shown in Figure 1-3. Connect the “ground” lead to the negative side of the resistor (point $A$).

2. Open the experiment file **Signal Filtering L10A1-3**.

3. Make sure that the computer signal generator is in manual mode (“unclick” the **Auto** button on the signal generator). Click on **Off** to turn off the computer signal generator. Turn on the PASCO PI-9587C Digital Function Generator-Amplifier and set the frequency to 500 Hz (If fed to a speaker, this signal would produce a tone slightly higher than a natural “A” which is 440 Hz).

4. Press **Start** to begin taking data. You will now be observing on Scope 1 the voltage across the resistor.

5. Use the **Smart Tool** and adjust the amplitude of the PASCO PI-9587C Digital Function Generator-Amplifier to 3.0 V on the computer screen, not on the generator. Unplug the cable that goes to the other signal generator to make sure its signal is not interfering with what you are doing. Adjust the time scale on the scope to 0.5 ms/div. After you are finished, then connect the cables back to the other signal generator.

6. Set the computer signal generator to 3.0 V at 4000 Hz and turn it on. On the scope you should see a large sine wave with a small ripple superimposed on top. Stop the computer and print the waveform, label it as Resistor and include one copy in your group report.

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Figure 1-3: Capacitive and Inductive Filter Circuit

- $R = 33 \, \Omega$
- $L = 8.2 \, \text{mH}$
- $C = 1.23 \, \mu\text{F}$
Question 1-11: How is the sine wave with the small ripple superimposed on top produced? If you were to turn off the computer signal generator, what would Scope 1 display? If you were to turn off the PASCO PI-9587C Digital Function Generator-Amplifier, what would Scope 1 display? Don’t actually do this – just try to answer the question.

Prediction 1-3: Imagine moving the ground connection to the negative side of the capacitor (point C) in Figure 1-3, what do you think the output on the scope would look like? How about for the inductor (point B)? Sketch your predictions for each in the appropriate box below including scale factors for voltage and time.

Capacitor

Inductor

7. Test your previous predictions by moving the ground connection from the resistor to the capacitor (point C) and then to the inductor (point B). Print out each waveform and attach them to your report. Make sure to label each printout as either Capacitor or Inductor.

Imagine that you replace the signal generator in the circuits in Figures 1-1 and 1-2 with a collection of different frequency signals coming from your CD player. You also have a woofer and a tweeter. You can connect either across the resistor in one of the circuits, in place of the voltage probe.
Question 1-12: Into which circuit should you wire the woofer--the capacitive filter circuit or the inductive filter circuit? (Note: assume that the bigger the voltage across the resistor, the louder the sound emitted by the speaker.) Briefly explain your reasoning.

Question 1-13: Into which circuit should you wire the tweeter--the capacitive filter circuit or the inductive filter circuit? Briefly explain your reasoning.

Investigation 2: The Series RLC Resonant (Tuner) Circuit

In this investigation, you will use your knowledge of the behavior of resistors, capacitors and inductors in circuits driven by various AC signal frequencies to predict and then observe the behavior of a circuit with a resistor, capacitor, and inductor connected in series.

The RLC series circuit you will study in this investigation exhibits a “resonance” behavior that is useful for many familiar applications. One of the most familiar uses of such a circuit is as a tuner in a radio receiver.

You will need the following materials:

- voltage probe
- RLC Circuit Board
- PASCO signal generator

Consider the series RLC circuit shown in Figure 2-1 (below).
**Prediction 2-1:** At very low signal frequencies (near 0 Hz), will the maximum values of $I$ through and $V$ across the resistor be relatively large, intermediate or small? Explain your reasoning.

**Prediction 2-2:** At very high signal frequencies (well above 3000 Hz), will the maximum values of $I$ and $V$ be relatively large, intermediate or small? Explain your reasoning.

**Prediction 2-3:** Based on your Predictions 2-1 and 2-2, is there some intermediate frequency where $I$ and $V$ will reach maximum or minimum values? Do you think they will be maximum or minimum?
1. On the axes below, draw qualitative graphs of $X_C$ vs. frequency and $X_L$ vs. frequency. Remember that 
\[ X_C = \frac{1}{\omega C} = \frac{1}{(2\pi fC)} \] and 
\[ X_L = \omega L = 2\pi fL. \]
Clearly label each curve.

2. For what relative values of $X_L$ and $X_C$ will the total impedance of the circuit, $Z$, be a minimum? On the axes above, mark and label the frequency where this occurs. Explain your reasoning here.

**Comment:** As we noted earlier in this lab, the relationship between the total impedance, $Z$, for a series combination of a resistor, capacitor, and inductor is not just the sum of the impedances of the three circuit elements. Instead, because of phase differences, $Z$ is given by the following expression:
\[ Z = \sqrt{R^2 + (X_L - X_C)^2} \]
Question 2-1 At the frequency you labeled, will the value of the peak current, \( I_{\text{max}} \), in the circuit be a maximum or minimum? What about the value of the peak voltage, \( V_{\text{max}} \), across the resistor? Explain.

Note: The point you identified in step 2 is the resonant frequency. Label it with the symbol \( f_0 \). The resonant frequency is the frequency at which the impedance of the series combination of a resistor, capacitor and inductor is minimal. This occurs at a frequency where the values of \( X_L \) and \( X_C \) are equal.

3. On the axes above (after step 1) draw a curve that qualitatively represents \( X_L - X_C \) vs. frequency. Be sure to label it.

4. Use your results from above to determine the general mathematical expression for the resonant frequency, \( f_0 \), as a function of \( L \) and \( C \). (Hint: you will need the expressions for \( X_C \) and \( X_L \) given to you in step 1)

Equation for \( f_0 \) : ___________________________

You will now test your predictions.

Activity 2-1: The Resonant Frequency of a Series RLC Circuit.

1. Open the experiment file RLC Resonance L10A2-1.

2. Adjust the scope display to 1 V/div and 1 ms/div.

3. Connect the circuit with resistor, capacitor, inductor, internal signal generator and probe shown in Figure 2-1.

4. Use the internal signal generator and set it to a frequency of 200 Hz and amplitude of 5 V.

5. Press Start to begin taking data

6. Use the Smart Tool to determine the peak voltage, \( V_{\text{max}} \).

7. Enter the data in the first row of Table 2-1.
Table 2-1

<table>
<thead>
<tr>
<th>frequency (Hz)</th>
<th>$V_{\text{max}}$ (V)</th>
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<tbody>
<tr>
<td>200</td>
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<td>700</td>
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<td>2700</td>
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<td>3200</td>
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</table>

8. Repeat steps 5 through 7 for the other frequencies in Table 2-1. Be sure that the amplitude of the signal generator is always 5 V.

9. Calculate the resonant frequency for your circuit. Show your calculations. Use the formula from step 4 and the actual values of the capacitance and inductance.)

$$f_{\text{resonance}} = \text{______________ Hz \hspace{1cm} CALCULATED}$$

10. Measure the resonant frequency of the circuit to within a few Hz. To do this, press Start to begin taking data and slowly adjust the frequency of the signal generator until the peak voltage across the resistor is maximal. It may be helpful to use the scope display for this. (Use the results from Table 2-1 to help you locate the resonant frequency.)

$$f_{\text{resonance}} = \text{__________ Hz \hspace{1cm} EXPERIMENTAL}$$

**Question 2-2:** How does this experimental value for the resonant frequency compare with your calculated one?

**Activity 2-2: Phase in an RLC Circuit**

In previous labs (and in this one), you investigated the phase relationship between the current and voltage in an AC circuit composed of a signal generator connected to one of the following circuit elements: a resistor, capacitor, or an inductor. **You found that**
the current and voltage are in phase when the element connected to the signal generator is a resistor, the current leads the voltage with a capacitor, and the current lags the voltage with an inductor.

You also discovered that the reactances of capacitors and inductors change in predictable ways as the frequency of the signal changes, while the resistance of a resistor is constant – independent of the signal frequency. When considering relatively high or low signal frequencies in a simple RLC circuit, the circuit element (either capacitor or inductor) with the highest reactance is said to “dominate” because this element determines whether the current lags or leads the voltage. At resonance, the reactances of capacitor and inductor cancel, and do not contribute to the impedance of the circuit. The resistor then is said to dominate the circuit.

In this activity, you will explore the phase relationship between the applied voltage (signal generator voltage) and current in an RLC circuit.

Consider the RLC circuit shown below.

![Figure 2-2: RLC series circuit](image)

**Question 2-3:** Which circuit element (the resistor, inductor, or capacitor) dominates the circuit in Figure 2-2 at frequencies well below the resonant frequency? Explain.
Question 2-4: Which circuit element (the resistor, inductor, or capacitor) dominates the circuit in Figure 2-2 at frequencies well above the resonant frequency? Explain.

Question 2-5: In the circuit in Figure 2-2, will the current through the resistor always be in phase with the voltage across it, regardless of the frequency? Explain your reasoning.

Question 2-6: In the circuit in Figure 2-2, will the current through the resistor always be in phase with applied voltage from the signal generator? Why or why not?

Prediction 2-4: If your answer to Question 2-6 was no, then which will lead for frequencies below the resonant frequency (current or voltage)? Which will lead for frequencies above the resonant frequency (current or voltage)?
Test your predictions.

1. Open the experiment file called **RLC Phase L10A2-2**.

2. Connect the circuit shown in Figure 2-2.

3. Set the signal generator to a frequency 200 Hz below the resonant frequency you measured in Activity 2-1, and set the amplitude of the signal to 5 V.

4. Press **Start** to begin taking data.

5. Determine whether the current or applied voltage leads.

**Question 2-7:** Which leads—applied voltage, current or neither—when the AC signal frequency is lower than the resonant frequency? Were your predictions correct? Why or why not? Explain.

6. Set the signal generator to a frequency 200 Hz above the resonant frequency with the amplitude of signal still 5 V.

7. Determine whether the current or applied voltage leads.

**Question 2-8:** Which leads—applied voltage, current or neither—when the AC signal frequency is higher than the resonant frequency? Were your predictions correct? Why or why not? Explain.

**Prediction 2-5:** Which will lead for an applied signal at the resonant frequency (current or voltage or neither)?
8. Set the signal generator to the resonant frequency you measured in Activity 2-1, and set the amplitude of the signal to 5 V.

9. Determine whether the current or applied voltage leads.

**Question 2-9:** At resonance, does the current or applied voltage lead (or neither)?

**Question 2-10:** Could you use this result to find the resonant frequency, if you did not already know it?

**Summary:** In Investigation 1, you learned how the frequency dependence of impedance for capacitors and inductors can be exploited to design filters to pass high or low frequency signals. Furthermore, you saw how this could be applied to the design of speakers. In this investigation you have investigated the phenomenon of resonance. One common application of the frequency-dependent impedances of inductors and capacitors is the series RLC resonant circuit. This circuit can be used, for example, as a radio tuning circuit.