

Homework #4 Solutions**Due: Friday September 25, 1998**

1 (Tipler 20-65). (a) The total energy of this system is in potential energy between the two charges at $t = 0$, and is given by

$$E_{\text{Total}} = \frac{(Q)(-Q)}{4\pi\epsilon_0[a - (-a)]}. \quad (1)$$

After the charges are released, both will pick up kinetic energy and hence will lose potential energy. The sum is constant (i. e. the energy is conserved),

$$2 \cdot \frac{1}{2}mv^2 - \frac{Q^2}{4\pi\epsilon_0(2x)} = -\frac{Q^2}{4\pi\epsilon_0(2a)} = E_{\text{Total}}. \quad (2)$$

Solve for v to obtain the velocity,

$$\mathbf{v}(x) = -\sqrt{\frac{Q^2}{8\pi\epsilon_0 m} \left(\frac{1}{x} - \frac{1}{a} \right)} \mathbf{i}. \quad (3)$$

The above expression is in the negative x -direction because we know that when the charges are released, they attract, meaning x decreases.

(b) To find $x(t)$, write $v \equiv dx/dt$ and separate the variables

$$-\int_a^0 \frac{dx}{\sqrt{\frac{1}{x} - \frac{1}{a}}} = \sqrt{\frac{Q^2}{8\pi\epsilon_0 m}} \int_0^t dt, \quad (4)$$

$$\sqrt{a} \int_0^a \sqrt{\frac{x}{a-x}} dx = \sqrt{\frac{Q^2}{8\pi\epsilon_0 m}} t. \quad (5)$$

Note that the limits on x are a to 0 since we know that the charges will collide at the origin. The integral on the left can (eventually) be found in the standard tables:

$$\int \sqrt{\frac{b-x}{d+x}} dx = \int \sqrt{\frac{-b+x}{-d-x}} dx = \sqrt{(b-x)(d+x)} + (b+d) \sin^{-1} \sqrt{\frac{d+x}{b+d}}. \quad (6)$$

This is what we want, where we have $b = 0$ and $d = -a$:

$$\int_0^a \sqrt{\frac{x}{a-x}} dx = \left(\sqrt{(-x)(-a+x)} + (-a) \sin^{-1} \sqrt{\frac{-a+x}{-a}} \right) \Big|_0^a = a \sin^{-1} 1 = \frac{\pi a}{2}. \quad (7)$$

We may now go back and evaluate the left side of eq. (5),

$$\frac{\pi a \sqrt{a}}{2} = \sqrt{\frac{Q^2}{8\pi\epsilon_0 m}} t, \quad (8)$$

and solve for the time to collision t ,

$$t = \sqrt{\frac{2\pi^3 \epsilon_0 m a^3}{Q^2}}. \quad (9)$$

2 (Tipler 21-39). (a) The capacitance for a parallel plate capacitor is given by

$$C = \frac{\kappa\epsilon_0 A}{d} = \frac{\kappa\epsilon_0 2\pi r L}{d}, \quad (10)$$

where the values are given by the problem statement. This gives $C = 1.7 \times 10^{-8}$ F, or 17,000 pF.

- (b) $Q = VC = 1.2 \times 10^{-9}$ C.
 (c) $E = V/d = 7 \times 10^6$ V/m.

3 (Tipler 21-60). (a) The whole of the upper plate (a conductor) is at some fixed potential, and the lower plate is also at some (different) fixed potential. Since the potential between the plates is $V = Ed$ no matter how you slice it, E must be the same throughout the volume between the plates, dielectric or not. Note that this potential difference between the plates may not be the same after the insertion of the dielectric as it was originally, since it is the total charge that remains constant. This would not be true if the battery were still attached to the capacitor.

(b) The electric field is reduced by a factor κ , so $E = \sigma/\kappa\epsilon_0$. Generally speaking, the κ and the ϵ_0 are buddies, enough so that a quantity called the permittivity is defined $\epsilon \equiv \kappa\epsilon_0$. Continuing on, we can derive the surface charge densities $\sigma_1 = \kappa\epsilon_0 E = 2\epsilon_0 E$, and $\sigma_2 = \epsilon_0 E$, so that $\sigma_1 = 2\sigma_2$.

(c) The total charge Q on each plate is constant, so we'll need to find σ_2 (or σ_1) in terms of the known values. Let's focus on the plate with the positive charge:

$$Q = \sigma_1 \cdot \frac{A}{2} + \sigma_2 \cdot \frac{A}{2} = \frac{3\sigma_2 A}{2}, \quad (11)$$

$$\downarrow$$

$$\sigma_2 = \frac{2Q}{3A}. \quad (12)$$

The new potential difference V' is given by

$$V' = Ed = \frac{\sigma_2 d}{\epsilon_0} = \frac{2Qd}{3\epsilon_0 A}. \quad (13)$$

The new capacitance C' is therefore

$$C' = \frac{Q}{V'} = \frac{3\epsilon_0 A}{2d}, \quad (14)$$

which is 3/2 what it was before the insertion of the dielectric. The new potential $V' = Q/C' = Q/(3C/2) = 2Q/3C$ is 2/3 the original potential $V = Q/C$.

4 (Tipler 22-50). The power dissipated in an ohmic material like copper is given by

$$P = I^2 R = I^2 \frac{\rho L}{\pi r^2}, \quad (15)$$

where we have already assumed a circular cross section of radius r for the wire. Solve this for r

$$r = \sqrt{\frac{\rho I^2}{\pi(P/L)}}, \quad (16)$$

where the conductivity of copper $\rho = 1.7 \times 10^{-8} \Omega \cdot \text{m}$, $I = 20$ A, and $P/L = 2$ W/m. This gives approximately 2.1 mm for the diameter of the wire.

5 (Tipler 22-62). (a) The total charge accelerated in each pulse is $Q = I\Delta t = (1.6)(0.1 \times 10^{-6}) = 1.6 \times 10^{-7}$ C, which means there are $N = Q/e = 10^{12}$ electrons, where e is the charge of an electron.

(b) Pulses repeated at a rate of 10^3 per second implies that there are 10^{-3} s = 1 ms between pulses. The average current is therefore the total charge accelerated over the whole period of repetition, formally,

$$I_{\text{ave}} = \frac{\int_0^T I(t) dt}{T} = \frac{Q \text{ moved in time } T}{T}. \quad (17)$$

This, however, is a hammer too big for this particular nail. We can calculate simply, $I_{\text{ave}} = Q/(10^{-3} \text{ s}) = 0.16$ mA.

- (c) The average power would be given by $P_{\text{ave}} = I_{\text{ave}}V = (0.16 \text{ mA})(400 \times 10^6 \text{ V}) = 64 \text{ kW}$.
- (d) The peak power output is the amount of power the accelerator is putting out when the pulse is on, so $P_{\text{peak}} = I_{\text{peak}}V = (1.6 \text{ A})(400 \times 10^6 \text{ V}) = 640 \text{ MW}$.
- (e) The duty cycle is simply the pulse width divided by the time between pulses, or $0.1 \mu\text{s}/1 \text{ ms} = 10^{-4}$, a result that is by this time anti-climactic. They can't all be winners, folks.