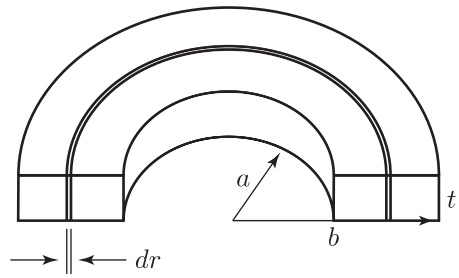
Homework #5 Solutions
Due: Friday October 9, 1998

1 (Tipler 22-65).



Since we know that the resistance of a wire is proportional to its length, current will preferentially flow at the inner radius. To follow through with this physically, imagine that the ring is cut up into little semicircular slices of constant radius r and depth Δr. It's resistance is given by

$$\Delta R_{\text{strip}} = \frac{\rho(\pi r)}{(t\Delta r)}, \tag{1}$$

where we have inserted its length $L = \pi r$ and it's cross-sectional area $A = t\Delta r$. Any given strip will be in parallel (circuit-wise) with any other strip, since both ends of the whole ring are held at some constant potential. To get the resistance of the whole ring, we need to use the formula for resistors in parallel:

$$\frac{1}{R} = \sum_{\text{all strips}} \frac{1}{\Delta R_{\text{strip}}} = \sum_{\text{all strips}} \frac{t\Delta r}{\pi\rho r}. \tag{2}$$

In the limit $\Delta r \to 0$, we can make the replacements $\Delta r \to dr$ and $\sum \to \int$ to obtain

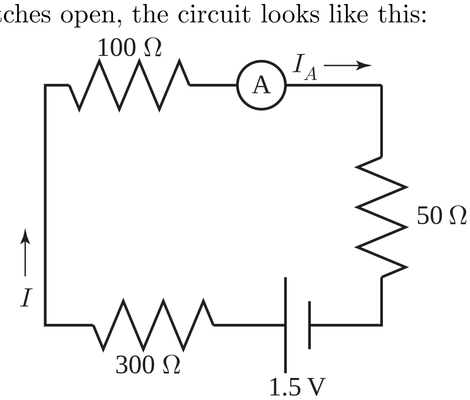
$$\frac{1}{R} = \frac{t}{\pi\rho}\int_a^b \frac{dr}{r} = \frac{t}{\pi\rho}\ln\frac{b}{a}. \tag{3}$$

The resistance of the ring is therefore $R = \pi\rho/t\ln(b/a)$. Note that there are incorrect ways to do this problem that involve "quantities" such as

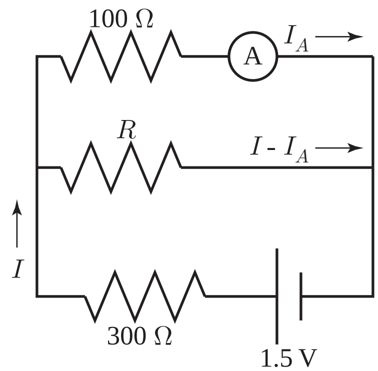
$$\int \frac{1}{dr}. \tag{4}$$

These don't make sense.

2 (Tipler 23-30). With both switches open, the circuit looks like this:



This is simply 3 resistors in series, so the current I_A in the meter is the same as the total current I flowing out of the battery, and is given by $I_A = V/R_{\text{series}} = (1.5)/(300 + 100 + 50) = 1/300\ A$. When the switches are closed, a resistance R is placed in parallel with the 100 Ω resistor and the 50 Ω resistor is shorted out, leaving this circuit:



We are given that I_A hasn't changed, but that doesn't mean that I hasn't changed. There are two routes to finding the value of R:

(clever) Since we know I_A, we can find the voltage drop across the 100 Ω, which is $\Delta V = I_A(100\ \Omega) = 1/3\ V$. This means the voltage drop across the 300 Ω resistor must be $1.5 - \Delta V = 7/6\ V$, and the total current must be $I = (7/6)/(300) = 7/1800$ A (yes, these fractions are getting ridiculous, but let's try to avoid rounding errors). The current through R must be $I - I_A$, so we have

$$R = \frac{\Delta V}{I - I_A} = \frac{1/3}{1/300 - 7/1800} = 600\ \Omega. \tag{5}$$

(by the book) We've already followed Kirchhoff's 2nd law in writing the currents for this circuit. We therefore need to write down two equations in the two unknowns I and R. Using Kirchhoff's 1st law, we can write down three:

$$0 = -1.5\ \text{V} + (300\ \Omega)I + (100\ \Omega)I_A, \tag{6}$$
$$0 = -1.5\ \text{V} + (300\ \Omega)I + (I - I_A)R, \tag{7}$$
$$0 = (100\ \Omega)I_A - (I - I_A)R. \tag{8}$$

These are the voltage loop equations for the (6) outermost, (7) lower, and (7) upper loops. Any two of these can be used to solve for R and I (probably (6) and (8) are best), and will give you the correct answer as well as the previous method does.

3 (Tipler 23-48). (a) Remember that the value $R_c \equiv V/I$ is defined not by the emf $\mathcal{E}$ or the total current flowing out of the battery, but rather by the values V, the voltage read by the voltmeter, and I, the current read by the ammeter. In the first circuit, the current flowing through the ammeter is the sum of the currents flowing through the resistor R and the voltmeter together (i. e. in parallel). The current it measures is therefore $I = I(\text{through R}) + I(\text{through voltmeter}) = V/R + V/R_v$. We then get that $1/R_c = I/V = 1/R + 1/R_v$. In the second circuit, the ammeter reads the current through R only, but now the voltmeter reads the voltage drop across both R and the ammeter. So $V = V(\text{across R}) + V(\text{across ammeter}) = IR + IR_a$. Again, $R_c = V/I = R + R_a$.
(b) Having R_c within 5% of R means that

$$0.95 \le \frac{R_c}{R} \le 1.05, \tag{9}$$

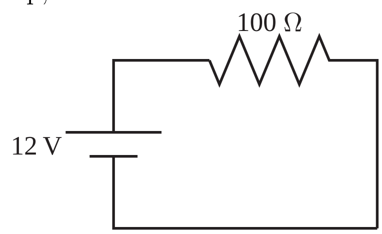
and means (almost) the same thing as

$$0.95 \le \frac{R}{R_c} \le 1.05, \tag{10}$$

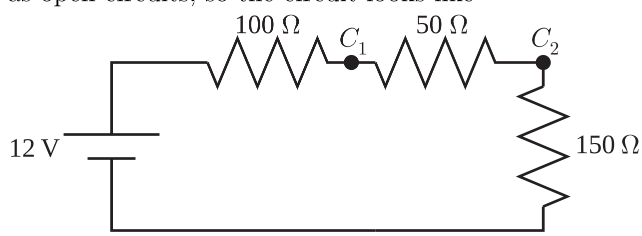
but either is good enough for our nefarious purposes. We can rewrite the result for the first circuit as $R/R_c = 1 + R/R_v$, which brings us to the important point that R will always be greater than our value R_c. Only one side of the dual inequality makes sense in such a case, so we have $1 + R/R_v \le 1.05$, or $R \le 500\ \Omega$. If this isn't true, then too much current will be flowing through the voltmeter.

(c) The rewritten result for the second circuit is $R_c/R = 1 + R_a/R$. The opposite situation to part (a) is now encountered, where R will always be less than our value R_c. Again, only one side of the dual inequality is physical, so $1 + R_a/R \leq 1.05$ or $R \geq 0.2\ \Omega$. If this isn't true, then there will be too large of a voltage drop across the ammeter.

4 (Tipler 23-64). (a) When S_1 is first closed, no charge has built up on the capacitor C_1. By $V = Q/C$, there is no potential drop across the cap, i. e. it is a short. The circuit is simply



The current is just $I = (12\ \text{V})/(100\ \Omega) = 0.12$ A.
(b) A long time means that the caps have had time to charge all the way. This means that they no longer conduct and therefore act as open circuits, so the circuit looks like



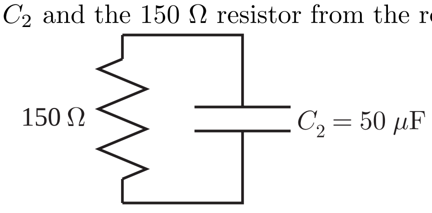
The current is then $I = (12\ \text{V})/(100\ \Omega + 50\ \Omega + 150\ \Omega) = 0.04$ A.
(c) & (d) Since C_1 is an open circuit, it is at the same potential as the point marked C_1 in the above figure. This is given by the so-called voltage divider equation,

$$V(C_1) = IR_2 + IR_3 = \frac{V}{IR_1 + IR_2 + IR_3}(R_2 + R_3) = V\left(\frac{R_2 + R_3}{R_1 + R_2 + R_3}\right), \tag{11}$$

where R_1, R_2, and R_3 are defined as you would expect. The voltage at C_1 is then $V(C_1) = (12) \cdot (50 + 100)/(100 + 50 + 100) = 8$ V. The voltage at C_2 is found the same way,

$$V(C_2) = IR_3 = \frac{VR_3}{IR_1 + IR_2 + IR_3} = V\left(\frac{R_3}{R_1 + R_2 + R_3}\right), \tag{12}$$

which is 6 V.
(e) When S_2 is opened, it isolates C_2 and the 150 Ω resistor from the rest of the circuit.



This is just an RC circuit with time constant $RC = (150\ \Omega)(50\ \times\ 10^{-6}\ \text{F}) = 0.0075$ s. The initial current that flows is $I_0 = V_0/R = (6\ V)/(150\ \Omega) = 0.04$ A, where we have used the result from part (d). The current as a function of time is

$$I(t) = I_0 e^{-t/RC} = (0.04\ \text{A}) \cdot e^{-t/(0.0075\ \text{s})}. \tag{13}$$

5 (Tipler 24-48). (a) The speed of the ions is given by the velocity selector equation

$$v = \frac{E}{B} = \frac{160\ \text{V}/(0.002\ \text{m})}{0.42\ \text{T}} = 1.9\ \times\ 10^5\ \text{m/s}. \tag{14}$$

(b) The radius of the orbit of a charged particle in a magnetic field is given by $r = mv/qB$. In the case of a singly ionized atom, $q = e = 1.602 \times 10^{-19}$ C, and we have both v and B. Let's go:

$$\begin{aligned} r(\mathrm{U}^{238}) - r(\mathrm{U}^{235}) &= \frac{(m_{238} - m_{235})v}{eB}, && (15)\\ &= \frac{m_{235}v}{eB}\left(\frac{m_{238}}{m_{235}} - 1\right), && (16)\\ &\simeq \frac{(3.903 \times 10^{-25})(1.9 \times 10^{5})}{(1.602 \times 10^{-19})(1.2)}\left(\frac{238}{235} - 1\right), && (17)\\ &= 4.94 \times 10^{-3} \text{ m}. && (18) \end{aligned}$$

So the difference in diameters is $2\Delta r = 9.9 \times 10^{-3}$ m.