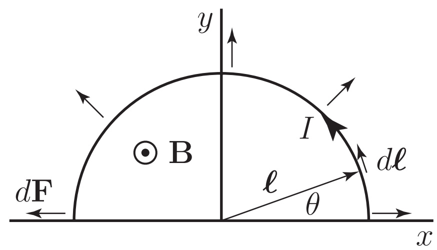
Homework #7 Solutions
Due: Friday October 23, 1998

1 (Tipler 24-54).



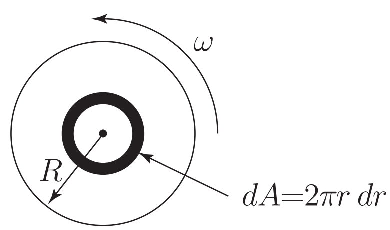
The current carrying wire follows a curve C described by the vector $\boldsymbol{\ell} = R\cos\theta\,\hat{\mathbf{i}} + R\sin\theta\,\hat{\mathbf{j}}$, where the parameter θ happens to be an angle and goes from zero to π. The force $d\mathbf{F}$ on each segment $d\boldsymbol{\ell}$ is given by $d\mathbf{F} = Id\boldsymbol{\ell} \times \mathbf{B}$. It is important to break up the problem in this fashion because, for example, the force on the right end $[(x, y) = (R, 0)$ or $\theta = 0]$ is to the right whereas the force on the left end $[(x, y) = (-R, 0)$ or $\theta = \pi]$ is to the left. Let's find the expression for $d\mathbf{F}$ as a function of our parameter θ:

$$d\mathbf{F} = Id\boldsymbol{\ell} \times \mathbf{B} = I\begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ (-R\sin\theta) & (R\cos\theta) & 0 \\ 0 & 0 & B \end{vmatrix} = IRB\,d\theta\left(\cos\theta\hat{\mathbf{i}} + \sin\theta\hat{\mathbf{j}}\right). \tag{1}$$

Finding the total force on the current half-loop involves the evaluation of the integral

$$\mathbf{F} = \int d\mathbf{F} = I\int_C d\boldsymbol{\ell} \times \mathbf{B} = IRB\left(\hat{\mathbf{i}}\int_0^\pi \cos\theta\,d\theta + \hat{\mathbf{j}}\int_0^\pi \sin\theta\,d\theta\right) = 2IRB\hat{\mathbf{i}}. \tag{2}$$

2 (Tipler 24-59).

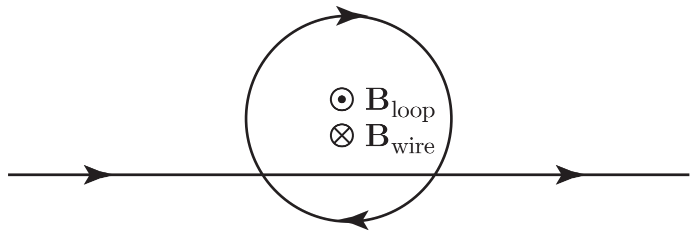


(a) The amount of charge in a ring of inner radius r and outer radius $r + dr$ is $dq = \sigma dA = \sigma(2\pi r\,dr)$, and passes through the same point once every period $T = 1/f = 2\pi/\omega$. The current is then $dI = dq/T = (\omega/2\pi)dq = \omega\sigma r\,dr$.
(b) The magnetic moment dm of a loop of current dI is $dm = dI\,A$, where A is the area inside the loop. This gives $dm = dI(\pi r^2) = \pi\omega\sigma r^3\,dr$.
(c) $m = \int dm = \pi\omega\sigma\int_0^R r^3\,dr = \pi\omega\sigma R^4/4$.
(d) The angular moment is given by (look it up) $\mathbf{L} = I\boldsymbol{\omega}$, where I for a solid disk is $MR^2/2$ and $\boldsymbol{\omega}$ has magnitude ω and is in the direction along the axis of rotation so that the sense of rotation is right-handed. This means that here, $\mathbf{L}$, $\boldsymbol{\omega}$, and $\mathbf{m}$ all point in the same direction. With that stated, we can just find the proportionality between m and L:

$$\frac{m}{L} = \frac{|\mathbf{m}|}{|\mathbf{L}|} = \frac{\pi\omega\sigma R^4/4}{MR^2\omega/2} = \frac{\pi\sigma R^2}{2M} = \frac{Q}{2M}, \tag{3}$$

where $Q = \sigma(\pi R^2)$ is the total charge on the disk. This proves the assertion that $\mathbf{m} = (Q/2M)\mathbf{L}$.

3 (Tipler 25-47).

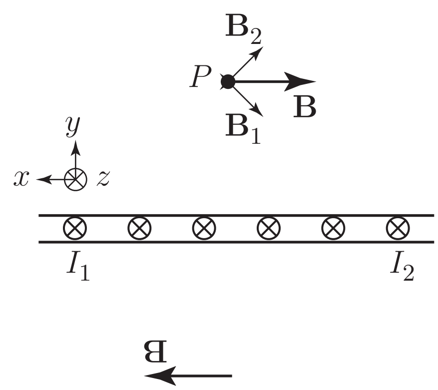


This problem is identical to the problems of finding the field due to a current loop and the field due to a long, straight wire. The field due to the loop is $B_{\text{loop}} = \mu_0 I/2R$ into the page, with $R = 10$ cm. The field due to the long wire is $B_{\text{wire}} = \mu_0 I/2\pi r$, and points out of the page at the center of the loop. The total field at the center of the loop is therefore zero if the fields due to the loop and the wire are equal in magnitude:

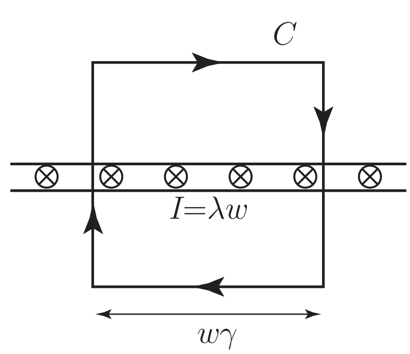
$$\begin{aligned} B_{\text{loop}} &= B_{\text{wire}} && (4)\\ \frac{\mu_0 I}{2R} &= \frac{\mu_0 I}{2\pi r} && (5)\\ r &= \frac{R}{\pi}, && (6) \end{aligned}$$

or $r = 3.2$ cm.

4 (Tipler 25-56).



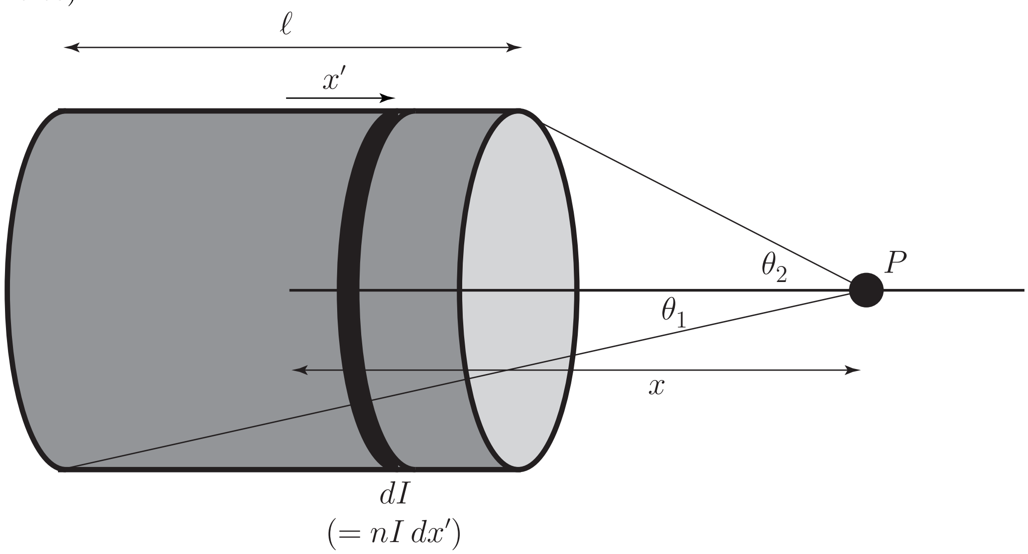
(a) The magnetic field due to I_1 is to the right and down, and the field due to I_2 is to the right and up. Since I_1 and I_2 are equidistant from P, the y-components cancel leaving only an x-component, which is to the right.
(b) The magnetic field due to the entire sheet is also to the right. Every small current dI to the left of P has a corresponding current to the right. Since the whole sheet is made up of such pairs, the field must be in the same direction that would be given from one pair.
(c) Below the sheet, we can apply the same reasoning as before to show that the magnetic field must be to the left.
(d)



Ampère's law states that $\int_C \mathbf{B} \cdot d\boldsymbol{\ell} = \mu_0 I_C$. Since we know that the magnetic field is to the right above the sheet and to the left below it, we know that $\mathbf{B} \cdot d\boldsymbol{\ell}$ along the sides of the curve must be 0 ($\mathbf{B}$ and $d\boldsymbol{\ell}$ are perpendicular), and that $\mathbf{B}\cdot d\boldsymbol{\ell}$ along the top and bottom of the curve is just Bw each ($\mathbf{B}$ and $d\boldsymbol{\ell}$ are parallel in both cases). The amount of current in this loop is $+\lambda w$, and is positive because we chose the curve C to be clockwise. Ampère's law therefore gives us that $2Bw = \mu_0\lambda w$ or $B = \mu_0\lambda/2$.

Some folks are confused by the choice of coordinate system. Since the problem states that the current is in the $+z$-direction, this means that the z-axis is into the page. The problem also states that the point P is above the sheet and has a positive y-coordinate, meaning that the y-axis is up. In order for the coordinate system to be right-handed (which the problem assumes), the x-axis must be to the left. The field above the sheet is to the right, and is therefore in the $(-x)$-direction.

5 (Tipler 25-63).



We may approximate the solenoid as being made up of a large number of current loops. We already know the magnetic field along the central axis of a circular loop of current I and radius R:

$$B_{\text{loop}} = \frac{\mu_0 I R^2}{2\left(x^2+R^2\right)^{3/2}}. \tag{7}$$

A small segment of length dx' of the solenoid has current $dI = nI\,dx'$, where n is the number of turns per unit length. This segment produces a field

$$dB = \frac{\mu_0 n I R^2\,dx'}{2\left[(x-x')^2+R^2\right]^{3/2}}, \tag{8}$$

where $(x - x')$ is the distance to the field point P. We'll need to sum up all of the loops to obtain the total field:

$$B = \int B = \frac{\mu_0 n I R^2}{2}\int_{-\ell/2}^{\ell/2}\frac{dx'}{\left[(x-x')^2+R^2\right]^{3/2}}. \tag{9}$$

This has been, thus far, a careful calculation. When it comes to evaluation, we can substitute variables, such as $\xi \equiv x - x'$, so $d\xi = -dx'$, and we get

$$\begin{aligned} B &= \frac{\mu_0 n I R^2}{2}\int_{x+\ell/2}^{x-\ell/2}\frac{(-d\xi)}{\left[\xi^2+R^2\right]^{3/2}} = \frac{\mu_0 n I R^2}{2}\int_{x-\ell/2}^{x+\ell/2}\frac{d\xi}{\left[\xi^2+R^2\right]^{3/2}} = \frac{\mu_0 n I R^2}{2}\left.\frac{\xi}{R^2\sqrt{\xi^2+R^2}}\right|_{x-\ell/2}^{x+\ell/2} && (10)\\ &= \frac{\mu_0 n I}{2}\left(\frac{x+\ell/2}{\sqrt{(x+\ell/2)^2+R^2}} - \frac{x-\ell/2}{\sqrt{(x-\ell/2)^2+R^2}}\right) && (11)\\ &= \frac{\mu_0 n I}{2}(\cos\theta_1 - \cos\theta_2), && (12)\end{aligned}$$

where $\cos\theta_1$ and $\cos\theta_2$ are defined by the problem.