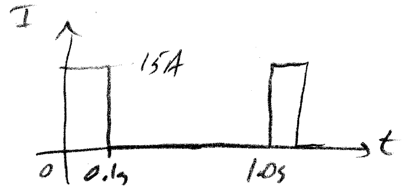
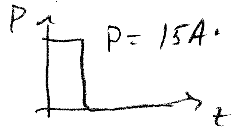
Homework #9
Tipler 28-36,44,54,59,70

36-



a) $I_{rms} = \sqrt{(15A)^2 \cdot \frac{0.1}{1.0}} = 4.7\ A$

b) $P = 15A \cdot 100V = 1500\ W$



$P_{ave} = 1500 \cdot \frac{0.1}{1.0} = \underline{150\ W}$

44.

$V_1 = 5.0V \cos(\omega t - \alpha)$

$V_2 = \underbrace{5.0V}_{V_0} \cos(\omega t + \alpha)$

$\alpha = \pi/6$

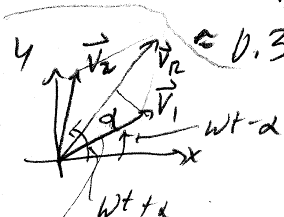
a) $I = \frac{V_1 + V_2}{R} = \frac{V_0}{R}\left[\cos(\omega t - \alpha) + \cos(\omega t + \alpha)\right]$

$= V_0/R \cdot 2 \cdot \cos\frac{(\omega t - \alpha) + (\omega t + \alpha)}{2} \cos\frac{(\omega t - \alpha) - (\omega t + \alpha)}{2}$

$= \frac{2V_0}{R} \cos\alpha \cos\omega t$

$\approx \underline{0.35\ A \cdot \cos\omega t}$

b)



$\vec{V}_1 + \vec{V}_2 = \vec{V}_R$ = Voltage drop across R

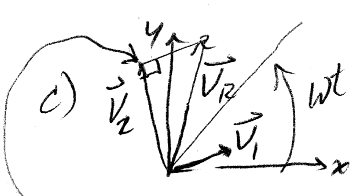
$|\vec{V}_1| = V_0$

$\frac{|\vec{V}_R|}{2} = V_0 \cos\alpha$

$|\vec{V}_R| = IR = 2V_0 \cos\alpha \Rightarrow I = \frac{2V_0}{R}\cos\alpha$

look at $\vec{V}_R \Rightarrow$ phase is clearly $\underline{\omega t}$

c)



right angle
cuz $\alpha = \frac{\pi}{4}$

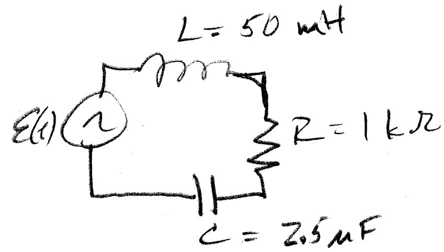
$|\vec{V}_R| = \sqrt{|\vec{V}_1|^2 + |\vec{V}_2|^2}$

$= \sqrt{5^2 + 7^2} = 8.6\ V \Rightarrow I = \frac{|\vec{V}_R|}{R} = 0.34\ A$

angle between $\vec{V}_R + \vec{V}_2 = \cos^{-1}\frac{|\vec{V}_2|}{|\vec{V}_R|} = 35.5° = 0.62\ rad = \theta$

$\Rightarrow I = 0.34A \cdot \cos(\omega t + 0.65\ rad)$

54 -



a) $f_{Resonance} = \frac{1}{2\pi} \cdot \frac{1}{\sqrt{LC}}$

$= 450 \text{ Hz}$

b) $Q = \frac{\omega_0 L}{R} = \frac{2\pi \cdot 450 \cdot 50 \text{ mH}}{1 \text{ k}\Omega} = 0.14$

c) $Q = \frac{\omega_0}{\Delta\omega} = \frac{f_0}{\Delta f}$ is <u>wrong</u> here because Q is too small

So, $P_{av} = \frac{\frac{1}{2}\mathcal{E}_0^2 R\omega^2}{L^2(\omega^2-\omega_0^2)^2 + \omega^2 R^2}$

Max. average Power $= \frac{\frac{1}{2}\mathcal{E}_0^2 R\omega^2}{\omega^2 R^2} = \frac{1}{2}\frac{\mathcal{E}_0^2}{R}$

So find $\frac{P_{av}}{P_{av\,max}} = \frac{1}{2} = \frac{\omega^2 R^2}{L^2(\omega^2-\omega_0^2)^2 + \omega^2 R^2}$

$L^2(\omega^2-\omega_0^2)^2 + \omega^2 R^2 = 2\omega^2 R^2$

$\frac{R}{2L} = 10^4$

$\omega_0^2 = 8 \times 10^6$

$L(\omega^2-\omega_0^2) = \pm \omega R$

$\omega^2 \mp \frac{R}{L}\omega - \omega_0^2 = 0 \Rightarrow \omega = \pm\frac{R}{2L} \pm \sqrt{\left(\frac{R}{2L}\right)^2 + \omega_0^2}$

$= \pm 10^4 \pm \sqrt{10^8 + 8\times 10^6}$

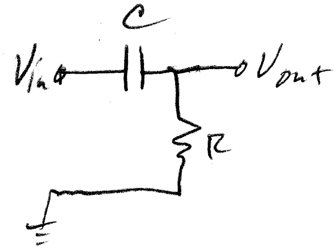
$= \pm 10^4 \pm 10{,}392$

$= \left.\begin{matrix} 392 \\ 20{,}392 \end{matrix}\right\}$ rad/s Answers $\Rightarrow f = \left\{\begin{matrix} 62 \text{ Hz} \\ 3{,}250 \text{ Hz} \end{matrix}\right.$

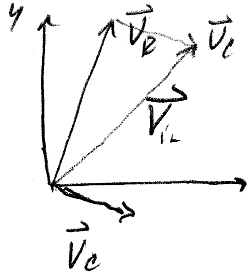
$\left.\begin{matrix} -392 \\ -20{,}392 \end{matrix}\right\}$ negative (freq > 0)

59.

a)



$V_{in} = V_0 \cos \omega t$



$\vec{V}_{in} = \vec{V}_C + \vec{V}_R$

$\vec{V}_{out} = \vec{V}_R$

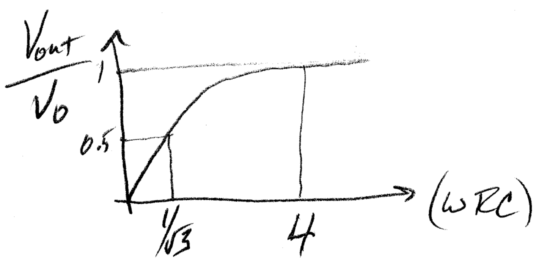
$V_{out} = IR$

$V_{in} = I \cdot Z_T \qquad Z_T = \sqrt{R^2 + \left(\frac{1}{\omega C}\right)^2}$

$$|\vec{V}_{out}| = V_{out} = IR = \frac{V_{in}}{Z_T} \cdot R = \frac{V_0 R}{\sqrt{R^2 + \left(\frac{1}{\omega C}\right)^2}} = \frac{V_0}{\sqrt{1 + \left(\frac{1}{\omega RC}\right)^2}}$$

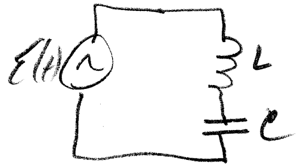
b) $V_{out} = \frac{V_0}{2} \Rightarrow 1 + \left(\frac{1}{\omega RC}\right)^2 = 4 \Rightarrow (\omega RC)^2 = \frac{1}{3} \Rightarrow \boxed{\omega = \frac{1}{RC\sqrt{3}}}$

c)



$$\frac{V_{out}}{V_0} = \frac{1}{\sqrt{1 + \frac{1}{(\omega RC)^2}}}$$

70 -



a) $\mathcal{E}(t) = \mathcal{E}_0 \cos\omega t = L\frac{dI}{dt} + \frac{Q}{C}$ say $I = \frac{dQ}{dt}$

$= L\frac{d^2Q}{dt} + \frac{Q}{C}$

b) $Q = Q_0 \cos\omega t$ a solution? check

$\frac{d^2Q}{dt} = -\omega^2 Q_0 \cos\omega t \Rightarrow L(-\omega^2 Q_0) + \frac{Q_0}{C} = \mathcal{E}_0$

$\Rightarrow Q_0 = \frac{\mathcal{E}_0}{\frac{1}{C} - \omega^2 L} = \frac{\mathcal{E}_0}{L(\frac{1}{LC} - \omega^2)}$

$= \frac{\mathcal{E}_0}{L(\omega_0^2 - \omega^2)} = -\frac{\mathcal{E}_0}{L(\omega^2 - \omega_0^2)}$

(whatever)

c) $I = \frac{dQ}{dt} = -\omega Q_0 \sin\omega t$

$= -\omega Q_0 \cos(\omega t - \frac{\pi}{2})$

$= \frac{\omega\mathcal{E}_0}{L(\omega^2 - \omega_0^2)} \cos(\omega t - \frac{\pi}{2})$

Want $I_0 > 0$

when $\omega > \omega_0$, $\frac{\omega\mathcal{E}_0}{L(\omega^2-\omega_0^2)} = I_0$ phase is $(\omega t - \frac{\pi}{2})$

$\omega < \omega_0$, $\frac{\omega\mathcal{E}_0}{L(\omega^2-\omega_0^2)} = -I_0$, phase is $(\omega t + \frac{\pi}{2})$

so $I = I_0 \cos(\omega t - \delta)$ where $I_0 = \frac{\omega\mathcal{E}_0}{L|\omega^2 - \omega_0^2|} = \frac{\mathcal{E}_0}{|X_L - X_C|}$

but $\delta = \begin{cases} \frac{\pi}{2} & \text{for } \omega > \omega_0 \\ -\frac{\pi}{2} & \text{for } \omega < \omega_0 \end{cases}$