Probabilities, Amplitudes and Probability Amplitudes

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Here we review three kinds of double slit experiments – *classical particles*, *classical waves*, and *quantum things*.

The lecture follows the very clear presentation in Feynman's *Lectures on Physics*.

Bullets

First we consider a double slit experiment with *bullets*. The source is a not very accurate machine gun, which sprays bullets towards two parallel slits, 1 and 2, in a sheet of armorplate, which may be covered one at a time with another piece of armor. The bullets are "detected" by plowing into a soft (but thick) wood screen some distance beyond the slits. We assume the bullets don't break up, you always find whole bullets in the wood.

If we now cover slit 2, and fire a lot of bullets, by examining the screen at the end we will see where the bullets went, and be able to construct a probability distribution $P_1(x)dx$ giving the probability that a bullet going through slit 1 lands in the interval $x, x + dx$. Similarly, covering slit 1, we can find the probability distribution $P_2(x)$ for bullets going through slit 2. Now suppose we open both slits—this gives a probability distribution $P_{12}(x)$ and it is clear that this will be simply the sum:

$$P_{12}(x) = P_1(x) + P_2(x).$$

This of course follows from the self-evident fact that any bullet getting to the screen must have gone either through slit 1 or slit 2. *The probabilities add*.

There is one other important point about bullets—they are lumpy. You either find one or you don't. This is also true of photons and electrons.

Water Waves

Now think about a double slit experiment with *waves*, say, surface waves on water—the standard ripple tank experiment. Assume that plane waves of definite wavelength impinge from the left on a barrier having two slits P_1 and P_2, with P_2 being initially closed. Then to the right of the barrier, waves emanate from the open slit with circular wavefronts. These waves are detected at a screen over to the right. The waves could be detected by having corks bobbing up and down in the water just in front of the screen. We could at some instant in time measure the height distribution $h_1(x)$ of the corks from waves coming through slit 1 (slit 2 being closed). Of course, this height distribution is changing all the time, and is not really what interests us. What we really want to know is the *intensity* of the wave motion at each point on the screen (analogous to the brightness of light, for example). *In any simple harmonic motion, the intensity is proportional to the square of the amplitude*. For our surface waves on water, the *amplitude* is of course just the height of the wave (relative to still water), so the energy intensity $I_1(x)$ for waves coming through slit 1 has the form

$$I_1(x) = Ah_1^2(x).$$

What happens when both slits are open? It is well known that waves superpose linearly—that is, the *heights* add. If at some instant of time with both slits open the height distribution of the corks is given by $h_{12}(x)$, then

$$h_{12}(x) = h_1(x) + h_2(x).$$

In this harmonic motion, the corks bob downwards as much as upwards, so h_1 and h_2 are as likely to cancel as they are to reinforce each other.

The energy intensity when both slits are open,

$$I_{12}(x) = A h_{12}{}^2(x) = I_1(x) + I_2(x) + A h_1(x) h_2(x).$$

The crucial point here is that in contrast to the bullets, the intensity when both slits are open is *not* the sum of the separate intensities for the slits being open one at a time. This is just the familiar wave interference effect.

Notice that in this classical wave picture, there is no lumpiness. The energy of the wave lapping against the screen varies smoothly, and can have any value we choose.

Classical Light

Historically, an experiment equivalent to the double slit experiment for light was what convinced people it was indeed a wave. The light reaching a screen exhibited the identical intensity pattern observed for water waves. This was fully explained with the advent of Maxwell's equations: the light was simply an electromagnetic wave, the amplitude of the electric field being analogous to the height of the water wave in the discussion above. To find the total electric field E_{12} at any point on the screen, the electric field E_1 of the wave emanating from S_1 is added to E_2 from S_2. The intensity of light I_{12} reaching the screen is proportional to $E_{12}{}^2$. Just as for the water waves, E_1 and E_2 oscillate about mean values of zero, so the analogy is complete.

Photons

The discovery that light is after all lumpy on a fine enough scale throws this picture into confusion. Experimentally, light of frequency f can only be detected in lumps of size hf.

To the classically trained mind, this naturally suggests that the source of light is actually sending out these lumps, and each lump presumably passes through one of the slits on its way to the screen. However, this naïve picture leads to a contradiction—how could it generate the observed diffraction pattern? Experimentally, the diffraction pattern builds up even if the light is so dim that there is a time interval between each photon (lump) passing through the apparatus. The picture of each photon going through one of the slits cannot be correct, because the distance between the slits governs the brightness pattern on the screen, and thus the probability pattern of where the photons land.

The only known resolution to this conceptual problem is to give up trying to visualize the path of the photon, just solve Maxwell's equation for the wave propagation, then interpret the resulting wave intensity as giving the relative probability of detecting the photon at any point. This is how quantum mechanics is done. It does not correspond to a physical picture readily interpretable in terms of familiar concepts. However, it does accord well with what is observed in nature.

Electrons

Electrons behave in the same way as photons; there is little to add to the description in the preceding section. The irony is, of course, that before quantum mechanics electrons were seen as bullets, light as waves, and it has turned out that they are *both* basic quantum particles, with their propagation governed by *wave* equations: but when they arrive, they act like *particles*!