

1. To get the mean free path of a molecule, we need the cross section and the number density. The latter can be obtained by noting that for 1 mole the perfect gas law gives the volume as

$$V = \frac{RT}{p}.$$

The pressure is 76 mm of Hg, meaning that it would support a column 0.76 m high. If the area of the column were 1 m² the mass of 0.76 m³ would be 10,336 kg. Thus the pressure is 101292.8 Nt/m². With a temperature of 150+273 °K we find the volume to be 34660 cm³. The number density is thus 1.74 × 10¹⁹/cm³. The cross section is roughly 12.6×10⁻¹⁶ cm² so the mean free path is 4.6×10⁻⁵ cm.

2. The mean time between collisions is obviously $\tau \approx \frac{\lambda}{\langle v^2 \rangle^{1/2}}$; the average velocity is given by

$$\langle v_x^2 \rangle = \frac{RT}{\mu}$$

where μ is the molecular weight in kilograms. Thus

$$\langle v_x^2 \rangle = \frac{8.3 \times 423}{0.029} = 12.1 \times 10^4 \text{ (m/sec)}^2$$

$$\text{or } \tau \approx 1.3 \times 10^{-9} \text{ sec.}$$

3. The surface tension of water at 100 °C is about 60 erg/cm². The heat of vaporization is about $H = 2.26 \times 10^{10}$ erg/gm. Assuming the water is separated into N cubes of side Λ , the total area is approximately $6N\Lambda^2$. On the other hand, the total volume is $N\Lambda^3$. Since one gm has a volume of 1 cm³, we see that

$$360N\Lambda^2 = 2.26 \times 10^{10}$$

$$N\Lambda^3 = 1 \text{ cm}^3$$

$$\text{hence } \Lambda \approx 1.6 \times 10^{-8} \text{ cm.}$$

4. I of course forgot to say what the bacterium's initial speed was. So let us consider the solution for arbitrary initial speed. The equation of motion is Newton's Second Law in one space dimension,

$$m \frac{dv}{dt} + \gamma v = f_{ext} = 0.$$

The solution is, noting

$$\frac{dv}{dt} = \frac{dv}{dx} \cdot \frac{dx}{dt} \equiv v \frac{dv}{dx},$$

$$v = v_0 - \frac{\gamma}{m} x$$

or

$$x_{stop} = \frac{mv_0}{\gamma}.$$

With $\gamma = 6\pi a\eta$ and $m = \rho \frac{4\pi}{3} a^3$ we find

$$x_{stop} = \frac{2}{9} \frac{\rho a^2 v_0}{\eta};$$

for initial speeds as great as

$$v_0 = 30 \mu\text{/sec}$$

we have $x_{stop} = 0.07 \text{ \AA}$ or less than 0.1 of an atomic radius.

5. The molar internal energy of the gas is

$$dU = \frac{f}{2} R dT;$$

if $dV = 0$ then

$$c_V = \left. \frac{df}{dT} \right|_{dV=0} = \frac{dU}{dT} = \frac{f}{2} R;$$

Conversely, if $dp = 0$ then

$$p dV = R dT \quad (\text{perfect gas law})$$

and so

$$c_p = \left. \frac{df}{dT} \right|_{dp=0}$$
$$= R + c_V = \left(1 + \frac{f}{2}\right)R.$$

For diatomic gases, the effective number of translational plus rotational degrees of freedom is 5; for polyatomic gases it is 6. Hence

$$\left. \begin{aligned} c_V &= 5/2R \\ c_p &= 7/2R \end{aligned} \right\}, \text{ diatomic}$$

$$\left. \begin{aligned} c_V &= 3R \\ c_p &= 4R \end{aligned} \right\}, \text{ polyatomic.}$$