Physics 312 – Assignment 1

Problem 1 – Questions C1-C5 on page 620 of *Tipler*

True or False: If the statement is true, explain why it is true. If it is false, give a counterexample.

(C1) The electric field of a point charge always points away from the charge.

False. This is only true for a positive point charge. The electric field of a negative point charge points toward the charge.

(C2) The charge of the electron is the smallest unit of charge found.

True. Every particle that has ever been isolated in a laboratory has a charge equal to an integer times the charge of an electron. (It is true that quarks carry charges that are smaller than that of an electron, but individual quarks have never been isolated.)

(C3) Electric field lines never diverge from a point in space.

False. Electric field lines diverge from a positive point charge.

(C4) Electric field lines never cross at a point in space.

True. If electric field lines crossed at a point in space then the electric field at that point would have two different values at the same time, which is impossible.

(C5) All molecules have electric dipole moments in the presence of an external electric field.

True. The external electric field polarizes the molecule, producing a dipole moment.

Problem 2 – Questions C1-C8 on page 652 of *Tipler*

True or False: If the statement is true, explain why it is true. If it is false, give a counterexample.

(C1) If there is no charge in a region of space, the electric field on a surface surrounding the region must be zero everywhere.

False. The flux through the surface must be zero, but the field itself need not be. Consider a point charge at the origin and a sphere of radius one meter located three meters from the origin. The sphere contains no charge, but the electric field is nonzero everywhere on its surface.

(C2) Gauss's law holds only for symmetric charge distributions.

False. Gauss's law holds for *any* charge distribution, but it is not usually useful for determining the electric field due to the charge distribution unless the distribution is symmetric.

(C3) The electric field inside a uniformly charged spherical shell is zero.

True. Because of symmetry we know that if there were an electric field inside the shell it would be radial. But there is no charge inside the shell, so Gauss's law gives $E_r = 0$. Therefore the entire electric field must be zero inside the uniformly charged spherical shell.

(C4) In electrostatic equilibrium, the electric field inside a conductor is zero.

True. Charges can move freely throughout a conductor, so if there were a nonzero

electric field anywhere inside the conductor the charges would quickly move to cancel it.

(C5) The result that $\mathbf{E} = 0$ inside a conductor can be derived from Gauss's law.

False. To show this result one must consider the physical characteristics of a conductor.

(C6) If the net charge on a conductor is zero, the charge density must be zero at every point on the surface of the conductor.

False. The *total* charge on the conductor must be zero, but there can be regions of positive charge so long as there are regions of negative charge to balance them.

(C7) The electric field is discontinuous at all points at which the charge density is discontinuous.

False. If the charge density is discontinuous then the *divergence* of the electric field must be discontinuous because of Gauss's law, but the field itself need not be. Consider, for example, the electric field of a solid sphere of charge (Figure 19-25 on page 643 of *Tipler*). The charge density is certainly discontinuous at the surface of the sphere, but the field itself is continuous.

(C8) Half of the electric field at a point just outside the surface of a conductor is due to the charge on the surface in the immediate vicinity of that point.

True. See page 647 of *Tipler* and especially Figure 19-32.

Problem 3 – Exercise 17 on page 621 of *Tipler*

Using Maple or the like, make a field plot and an equipotential plot for this charge configuration.

Figure 18-26 shows the electric-field lines for a system of two point charges.

(a) What are the relative magnitudes of the charges?

We see that the charge on the left has a total of 32 field lines intersecting it while the charge on the right has a total of 8 field lines intersecting it. It follows, then, that the charge on the left is four times the magnitude of the charge on the right.

(b) What are the signs of the charges?

The field lines diverge from the charge on the left and converge on the charge on the right. Therefore the charge on the left is positive and the charge on the right is negative.

(c) In what regions of space is the electric field strong? In what regions is it weak?

The electric field is strong near the charge on the left, and is especially strong in the area between the two charges. The electric field is weak far from the charges.

For the plots, we choose units so that the left charge has charge equal to $+4$ and is located at $(-1, 0, 0)$, while the right charge has charge equal to -1 and is located at $(1, 0, 0)$. Then the potential due to the two charges is (up to a multiplicative constant)

$$V(x,y) = \frac{4}{\sqrt{(x+1)^2 + y^2}} + \frac{-1}{\sqrt{(x-1)^2 + y^2}}.$$

We can then use the Maple functions `gradplot` and `contourplot` to make plots of the electric field and the equipotentials, respectively. The is done in the file `assignment1.mws`, which can also be found on the solutions webpage.

Problem 4 – Exercise 33 on page 622 of *Tipler*

An electron with a kinetic energy of $2 \cdot 10^{-16}$ J is moving to the right along the axis of a cathode ray tube as shown in Figure 18-29. There is an electric field $\mathbf{E} = \left(2 \cdot 10^4 \ \frac{\mathrm{N}}{\mathrm{C}}\right) \mathbf{j}$ in the region between the deflection plates. Everywhere else, $\mathbf{E} = 0$.

(a) How far is the electron from the axis of the tube when it reaches the end of the plates?

Call the initial kinetic energy of the electron K_i. Then

$$K_i = \frac{1}{2} m_e v_i^2,$$

where m_e is the mass of the electron and v_i is the initial speed of the electron. It follows, then, that the initial velocity of the electron is

$$v_i = \sqrt{\frac{2K_i}{m_e}}\mathbf{i}.$$

While between the plates the electron experiences a force $\mathbf{F} = e\mathbf{E} = eE\mathbf{j}$, where E is the magnitude of the electric field between the plates. Because this force does not act in the horizontal direction the horizontal component of the velocity is constant, and so the time taken by the electron to pass between the plates is given by

$$t_{plates} = \frac{x_{plates}}{v_i} = x_{plates}\sqrt{\frac{m_e}{2K_i}},$$

where x_{plates} is the horizontal extent of the plates. The vertical component of the velocity when the electron leaves the plates is then given by the basic kinematics equation $v = v_0 + at$ as

$$v_{vertical} = \left(\frac{F}{m_e}\right) t_{plates} = \frac{eE}{m_e} x_{plates}\sqrt{\frac{m_e}{2K_i}} = \frac{eE}{\sqrt{2m_e K_i}} x_{plates}.$$

The kinematics relation $v_f^2 = v_0^2 + 2ax$ yields

$$y_{plates} = \frac{v_{vertical}^2}{2\left(\frac{F}{m_e}\right)} = \frac{\frac{e^2E^2}{2m_eK_i}x_{plates}^2}{2\left(\frac{eE}{m_e}\right)} = \frac{eE}{4K_i}x_{plates}^2$$

for the distance the electron is from the axis of the tube when it reaches the end of the plates. Plugging in values, we find that

$$y_{plates} = \frac{\left(-1.602 \cdot 10^{-19}\mathrm{C}\right)\left(2 \cdot 10^4 \ \frac{\mathrm{N}}{\mathrm{C}}\right)}{4\left(2 \cdot 10^{-16} \ \mathrm{J}\right)}\left(4 \cdot 10^{-2}\mathrm{m}\right)^2 = -6.4 \cdot 10^{-3}\mathrm{m}.$$

Thus when the electron leaves the plates it is 6.4 mm below the axis of the tube.

(b) At what angle is the electron moving with respect to the axis?

The horizontal component of velocity is constant throughout the journey through the plates, so when the electron leaves the plates it is

$$v_{horizontal} = v_i = \sqrt{\frac{2K_i}{m_e}}.$$

We found in part (a) that the vertical component of the velocity when the electron leaves

the plates is

$$v_{vertical} = \frac{eE}{\sqrt{2m_e K_i}} x_{plates}.$$

Thus, when the electron leaves the plates it is travelling at an angle

$$\theta = \tan^{-1}\left(\frac{\frac{eE}{\sqrt{2m_e K_i}} x_{plates}}{\sqrt{\frac{2K_i}{m_e}}}\right) = \tan^{-1}\left(\frac{eE}{2K_i} x_{plates}\right)$$

with respect to the axis of the tube. Plugging in the values, we find that

$$\theta = \tan^{-1}\left(\frac{\left(-1.602 \cdot 10^{-19}\mathrm{C}\right)\left(2 \cdot 10^{4}\ \frac{\mathrm{N}}{\mathrm{C}}\right)}{2\left(2 \cdot 10^{-16}\ \mathrm{J}\right)}\left(4 \cdot 10^{-2}\mathrm{m}\right)\right) = -17.7^{\circ}.$$

Thus when the electron leaves the plates it is traveling at an angle 17.7° below the axis of the tube.

(c) How far from the axis will the electron be when it strikes the florescent screen?

After the electron exits the plates, both components of its velocity will remain constant because no force acts on the electron. The time between the electron exiting the plates and striking the florescent screen is then

$$t_{screen} = \frac{x_{screen}}{v_i} = x_{screen}\sqrt{\frac{m_e}{2K_i}},$$

where x_{screen} is the distance from the end of the plates to the screen. Because the vertical component of the velocity is constant during this time, the electron moves a distance

$$y_{screen} = v_{vertical} t_{screen} = \left(\frac{eE}{\sqrt{2m_e K_i}} x_{plates}\right)\left(x_{screen}\sqrt{\frac{m_e}{2K_i}}\right) = \frac{eE}{2K_i} x_{plates} x_{screen}.$$

Plugging in values, we see that

$$y_{screen} = \frac{\left(-1.602 \cdot 10^{-19}\mathrm{C}\right)\left(2 \cdot 10^{4}\ \frac{\mathrm{N}}{\mathrm{C}}\right)}{2\left(2 \cdot 10^{-16}\ \mathrm{J}\right)}\left(4 \cdot 10^{-2}\mathrm{m}\right)\left(12 \cdot 10^{-2}\ \mathrm{m}\right) = -3.84 \cdot 10^{-2}\mathrm{m}.$$

The total deflection in the horizontal direction is then

$$y_{plates} + y_{screen} = -4.48 \cdot 10^{-2}\mathrm{m}.$$

Therefore the electron strikes the screen at a distance $4.48\ \mathrm{cm}$ below the axis of the tube.