Physics 312 – Assignment 3

1. In a desktop electrostatic air cleaner, air is driven through the "cell" by a fan that draws $55\ W$ (when set at "high"). The cross sectional dimensions of the cell are $25\ \mathrm{cm}$ by $5\ \mathrm{cm}$. Neglecting losses, you can compute the speed of air flow, v, from these data. Compare this computed value of v with the value "measured" in class. (At the calculated v) how many minutes does it take to process a volume of air equal to that of a typical room? (Take the room to be 4 by 4 by 2 cubic meters).

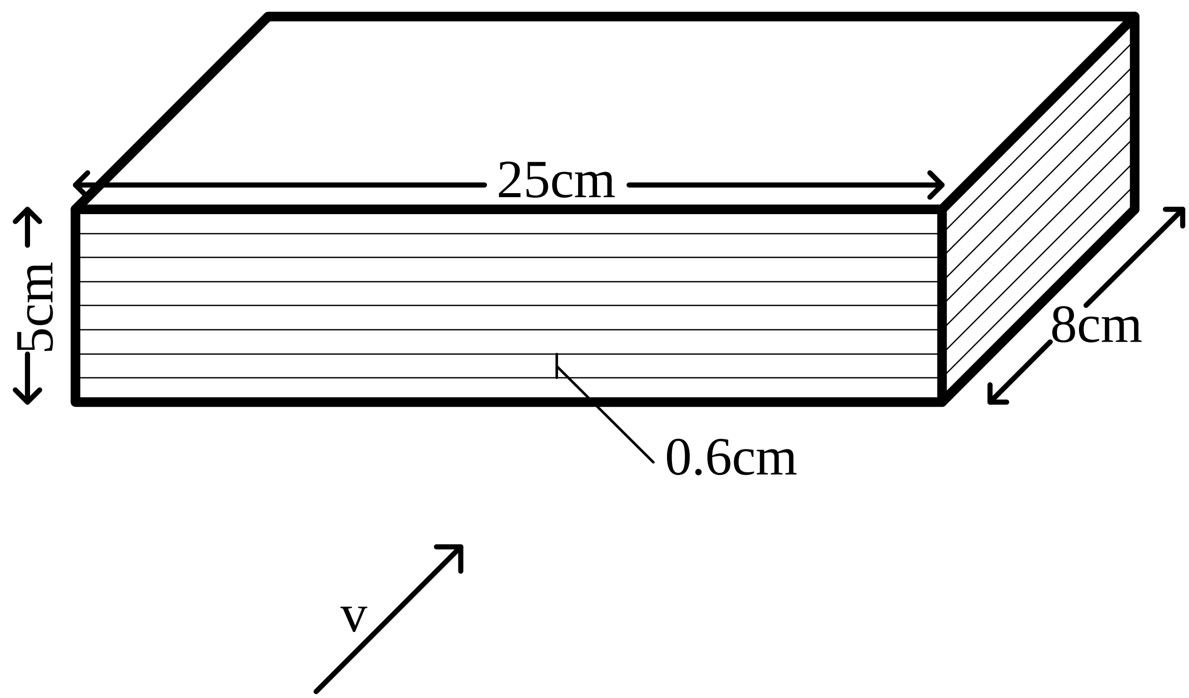

Figure 1:

After an initial period when the cleaner is first turned on, a steady stream of air is passing through it at a constant velocity. (To envision the "steady state" situation, it might help to think of the system as a long tube of length L and cross sectional area A that forms a closed circuit, but it's not necessary.) The air would come to a standstill rather quickly if we were not continually supplying it with energy, and the energy we must supply to the air is its (mechanical) kinetic energy (as opposed to its thermal kinetic energy). A volume of gas with dimensions ℓ by A has mass $m = \rho \ell A$, where ρ is the mass density of air.

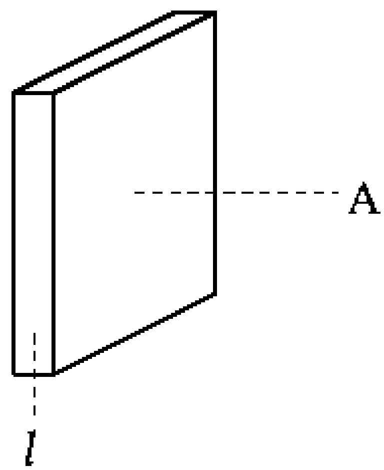

Figure 2:

We must supply to that volume of air an energy $\Delta E = \frac{1}{2} m v^2$, and the time in which we have to supply that energy is the time it takes for that volume to pass by the fan which is $\Delta t = \ell / v$. Therefore, the power, which is energy per time, is

$$P = \frac{\Delta E}{\Delta t} = \frac{\frac{1}{2}\rho \ell A v^2}{\ell / v} = \frac{\rho A v^3}{2}. \tag{1}$$

Solving for v yields

$$v = \left(\frac{2P}{\rho A} \right)^{1/3} \tag{2}$$

(Note that ℓ dropped out so that it could have been taken to be L, the imagined length of the entire system, or taken to be infinitesimally small.) Using that $\rho_{\mathrm{Air}} = 1.293\ \mathrm{kg/m^3}$ (from *Tipler*, p. 333) and the information above, we find that $v \approx 19\ \mathrm{m/s}$.

This velocity is somewhat high. It's equivalent to about $43\ \mathrm{miles/hour}$ *which would be a very high wind. So our velocity is probably* 2 *to* 3 *times too high. Thus not all of the power is going into translational kinetic energy. Where is it going?*

The product Av (which equals $0.24\ \mathrm{m^3/s}$) provides the volume of air passing by the fan per unit time. From it we can calculate the time needed to process a volume of $32\ \mathrm{m^3}$ to be $130\ \mathrm{s}$ or $2.2\ \mathrm{min}$ at this speed.

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2. Grains of pollen of mass M are entrained (carried along) by air moving at speed v, as calculated in problem 1, between plates held at a potential difference $\mathcal{V} = 5500$ Volts. The distance d between plates is 0.6 cm and the depth of a cell is 8 cm. Will the pollen be collected for sure on the positive plate when a grain picks up the elementary charge e or does it have to be multiply charged? I am particularly concerned with oak pollen that has very small grains, only 10 micrometers in diameter (you can estimate M from this or you can find out what M really is).

If we assume the pollen grains to have a denisty the same order of magnitude as water ($\rho_{\rm water} = 1 \times 10^3\ {\rm kg/m^3}$) and a spherical shape, then we can calculate the mass M from the diameter given above

$$M = \frac{4\pi}{3}\left(\frac{D}{2}\right)^3 \rho_{\rm water}, \tag{3}$$

which gives $M \approx 5.2 \times 10^{-13}$ kg. (*The factor* $4\pi/3$ *is overkill in such an order of magnitude calculation.*)

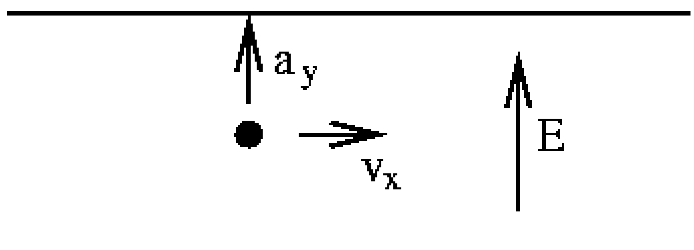

Figure 3:

Next we can calculate the acceleration due to the electric field the particle is passing through. The electric field between the plates is $E = \mathcal{V}/d$, where $\mathcal{V}$ is the voltage difference between plates and d the distance between plates. (We are neglecting edge effects.) Multiplying by the charge e gives the force, and dividing by M the acceleration

$$a_y = \frac{e\mathcal{V}}{Md}, \tag{4}$$

which yields $a_y = 0.28\ {\rm m/s^2}$. (*Note that we have neglected friction in this calculation as well as any turbulent motion of the air as it passes through the air cleaner.*)

Now we find the time it takes to pass through the plates as $t = s_x/v_x$ where v_x was calculated in the previous problem. We obtain $t = 4.2 \times 10^{-3}$ s. And finally we determine the deflection in the y direction from $s_y = \frac{1}{2}a_y t^2$ giving $s_y = 2.5 \times 10^{-6}$ m.

So a singly charged particle moves a couple micrometers, falling far short of the 0.6 cm it requires to ensure that it would hit the plate before leaving the air cleaner. We have suggested above that our calculation of v was too high, and this might change s_y by a factor of 10, but we need it to change by a factor closer to 10^3 to ensure trapping. Thus if particles of mass M estimated above are to be trapped, they must be multiply charged, with charges 100 to 1000 times that of a single electron.

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3. (Adapted from *Tipler*, page 1168.) True or false (explain briefly why, or comment briefly):

(3-1) The peak intensity of blackbody radiation varies linearly with the temperature of the body.

False. A quick inspection of Figure 35-2 on p. 1147 of *Tipler* shows that the peak intensity varies much faster than linearly.

(3-2) In the photoelectric effect, the current is proportional to the intensity of the incident light.

True. The current at any voltage is proportional to the number of photons arriving each second. (See also Figure 35-5 of *Tipler*.)

(3-3) The work function of a metal depends on the intensity of the incident light.

False. The work function of a metal is a characteristic of the particular metal, and has nothing to do with the intensity of the incident light. (*Tipler*, p. 1150)

(3-4) An electron microscope is used to look at electrons.

False. An electron microscope has a resolution of only a few tenths of a nanometer, and therefore it is impossible to see an electron with this device. (Figure 35-18 of *Tipler*)

(3-5) The shortest wavelength of the spectrum emitted by an x-ray tube is proportional to the voltage of the tube.

False. It is *inversely* proportional to the voltage. (*Tipler*, p. 1153)

(3-6) If an atom or molecule emits photons of a certain frequency, it will also absorb photons of that frequency.

True.

(3-7) In the ground state of the Hydrogen atom, the kinetic energy is 27.2 eV.

False. Let K be the kinetic energy of an electron in the state, let U be the potential energy of an electron in the state, and let $E = K + U$ be the total energy of an electron in the state. From equations 35-18 and 35-19 on page 1158 of *Tipler*, we have that $K = -\frac{1}{2}U$ and $E = \frac{1}{2}U$, respectively. Combining these two relations, we see that $K = -E$. Since the total, energy of an electron in the ground state of Hydrogen is -13.6 eV, we see that the kinetic energy is $K = 13.6$ eV.

(3-8) The de Broglie wavelength of a proton varies inversely with its momentum.

True. See equation 35-29 on p. 1161 of *Tipler*.

(3-9) The energy of a photon is proportional to its wavelength.

False. Equation 35-2 on p. 1148 of *Tipler* states that $E = hf$, and since $f = \frac{c}{\lambda}$, it follows that $E = \frac{hc}{\lambda}$.

(3-10) Electrons can be diffracted, neutrons can be diffracted, Helium atoms can be diffracted, benzene molecules can be diffracted, tennis balls can be diffracted, tennis shoes can be diffracted .

True for all in principle, but false for tennis balls and shoes in practice. Actually, until diffraction is demonstrated in practice with tennis balls, one can doubt whether quantum theory applies to such large objects.