Physics 312 – Assignment 6

1. Explain why a transient current flows when you touch a piece of n-type semiconductor to a piece of p-type semiconductor. What is the direction of current flow? What stops the current after a while? Similar questions are in *Bloomfield*, p. 439.

If we bring an n-type semiconductor (for instance, As - doped Ge) into contact with a p-type semiconductor (for instance, Ga - doped Ge), we have electrons in the conduction band on the n-type side and holes in the valence band on the p-type side. We can lower the energy by moving some of the former into the latter. Since the electrons are negatively charged, the current will be from the p-type side to the n-type side. There is also a concentration gradient, i.e. more electrons on one side, more holes on the other, which leads to a diffusion that from a region of high concentration to one of lower concentration. This effect pushes electrons in the same direction as the energy considerations. In fact, the two effects are related, because the carrier concentrations depend on the concentration of impurities, such as As and Ga,and their "donor" and "acceptor" levels (see below). The real quantity driving the migration of electrons is the "local chemical potential", which depends both on the "impurity levels" available and on the carrier concentrations.

Before this electron migration started both sides were electrically neutral, but now negative charges are building up on the p side and positive on the n side. It costs energy to separate charge. So when the gain in energy from moving electrons from conduction to valence bands balances the loss in energy from separating the charges, the current stops. (The current stops when the chemical potential is the same on both sides.)

This process creates a "depletion region" or "depletion zone," and it is important in the operation of diodes.

About n-type and p-type materials

A somewhat naive picture is as follows. Start with silicon or germanium (Ge) atoms which have four valence electrons. Take the Ge ions (the nuclei plus the bound electrons) and construct a lattice. Then find the states associated with this structure and fill them up (obeying the Pauli exclusion principle and starting with the lowest energy). One finds that the energies of these states fall into "bands" separated by "gaps." Furthermore, the last filled state completes a band (the valence band) and the next available state is E_g higher in energy.

Now let us consider substituting a small percentage of As (or P, or Sb) atoms for some of the Ge. Since As has five valence electrons, we have some extra electrons which could go in the higher (conduction) band. This isn't quite right because the As ions are different from the Ge ions (they have more charge); however, it is true that the Ge doped with As has very similar energy levels as pure Ge and that there are additional states, localized near the As impurities, that are close in energy to the conduction band of pure Ge. These are

called "donor" states. At ordinary temperatures, thermal energy is sufficient to promote electrons from the donor states to the conduction band. We end up with extra electrons in the conduction band ("carriers"), although their concentration is not the same as that of the As impurities, in general. This is called an n-type semiconductor.

If we substituted Ga atoms instead (or another atom with three valence electrons, such as B, Al, or In), the energy levels would again be quite similar to those in pure Ge, but this time there would be "acceptor" states associated with the Ga impurities, as well extra unfilled levels (holes) in the valence band at finite temperature. This is called a p-type semiconductor.

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2. *Melissinos*, exercise 1.1.

(a) Look up the atomic mass number A, and density ρ of Si and Ge and find the number of atoms per cm^3.

(b) Assuming that the atoms are in a diamond structure (8 atoms/unit cell) find the lattice spacing.

(c) Find the resistivity of Ge at room temperature if it is doped with 10^{15} $\mathrm{atoms/cm}^3$ of Sb. Assume a mobility of the donor's electrons of $\mu_e = 1200\ \mathrm{Cm}^2/\mathrm{V}\cdot\mathrm{s}$.

(a) From the CRC Handbook we find the mass numbers A and densities ρ for Si and Ge

$$\begin{aligned} A_{Si} &= 28.09 \qquad \rho_{Si} = 2.33\ \mathrm{g/cm}^3 \\ A_{Ge} &= 72.59 \qquad \rho_{Ge} = 5.32\ \mathrm{g/cm}^3 \end{aligned} \tag{1}$$

Noting that there are N_A atoms in a mole where N_A is Avogadro's number, we can calculate the number density from the mass density and the atomic mass, as follows

$$\begin{aligned} Si: \ \left(\frac{2.33\ \mathrm{g}}{\mathrm{cm}^3}\right)\left(\frac{6.022\times 10^{23}\ \mathrm{atoms}}{28.09\ \mathrm{g}}\right) &= 5.00\times 10^{22}\ \mathrm{atoms/cm}^3; \\ Ge: \ \left(\frac{5.32\ \mathrm{g}}{\mathrm{cm}^3}\right)\left(\frac{6.022\times 10^{23}\ \mathrm{atoms}}{72.59\ \mathrm{g}}\right) &= 4.41\times 10^{22}\ \mathrm{atoms/cm}^3. \end{aligned} \tag{2}$$

(b) Next, from the number density we can calculate the volume of a unit cell which contains 8 atoms in this instance

$$\begin{aligned} V_{cell-Si} &= 8\ \mathrm{atoms}\left(\frac{\mathrm{cm}^3}{5.00\times 10^{22}\ \mathrm{atoms}}\right) = 1.60\times 10^{-22}\mathrm{cm}^3; \\ V_{cell-Ge} &= 8\ \mathrm{atoms}\left(\frac{\mathrm{cm}^3}{4.41\times 10^{22}\ \mathrm{atoms}}\right) = 1.81\times 10^{-22}\mathrm{cm}^3. \end{aligned} \tag{3}$$

Then from the volume of the unit cell we can calculate the lattice spacing

$$\begin{aligned} a_{Si} &= \left(1.60\times 10^{-22}\ \mathrm{cm}^3\right)^{1/3} = 5.43\times 10^{-8}\ \mathrm{cm} = 5.43\ \mathrm{angstroms}; \\ a_{Ge} &= \left(1.81\times 10^{-22}\ \mathrm{cm}^3\right)^{1/3} = 5.66\times 10^{-8}\ \mathrm{cm} = 5.66\ \mathrm{angstroms}. \end{aligned} \tag{4}$$

(c) First of all we can see by the position of Antimony (Sb) on the periodic table that it is

a "donor" and that we are talking about an "n-type" (doped) semiconductor. The intrinsic carrier density n_i of Ge calculated at the end of Section 1.1 in *Melissinos* is $10^{13}/\ \mathrm{cm}^3$; it is much less than donor density N_D given in the problem as $10^{15}/\ \mathrm{cm}^3$. Therefore, the carrier density of negative charges n is essentially equal to N_D. The carrier density of positive charges is approximately n_i^2/N_D (see *Melissinos*, p. 11) which is very much smaller, and so we will neglect any contribution from positive carriers hereafter.

When only one carrier contributes the conductivity σ is given by

$$\sigma = qn\mu, \tag{5}$$

where q is the charge, n the carrier density, and μ the mobility (see *Melissinos*, p. 13). Thus

$$\sigma = \left(1.60 \times 10^{-19}\ \mathrm{C}\right)\left(10^{15}\ /\mathrm{cm}^3\right)\left(1200\ \mathrm{cm}^2/\mathrm{V}\cdot\mathrm{s}\right) = \frac{0.192\ \mathrm{C}}{\mathrm{V}\cdot\mathrm{s}\cdot\mathrm{cm}}. \tag{6}$$

Converting the cm to m and taking the reciprocal gives the resistivity $\rho = 5.21\times10^{-2}\quad \cdot\mathrm{m}$.

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3. *Melissinos*, exercise 1.3.

Consider germanium doped with $10^{14}/\mathrm{cm}^3$ atoms of arsenic.

(a) Find the conductivity assuming a reasonable value of the mobility of the impurities.

(b) The energy gap of germanium is $E_g = 0.67\ \mathrm{eV}$ and the density of states at the edge of the conduction band can be taken as $N_c = 10^{19}/\mathrm{cm}^3$. Estimate the intrinsic carrier density for germanium at room temperature.

(c) Use the result of (b) to find the density of holes in the doped sample.

(Note that part (a) is closely related to part (c) of exercise 1.1.)

(b) I think it's more sensible to answer part (b) first. The intrinsic carrier density n_i is related to the density of states at the edge of the conduction band N_C through

$$n_i = N_C \exp\left(-\frac{E_g}{2kT}\right). \tag{7}$$

(See *Melissinos*, p. 8.) So

$$n_i = 10^{19}/\mathrm{cm}^3 \times \exp\left(-\frac{0.67\ \mathrm{eV}}{2(\mathrm{eV}/40)}\right) \approx 1.5 \times 10^{13}/\mathrm{cm}^3. \tag{8}$$

(a) Arsenic (As) is a donor. The density of donors $N_D = 10^{14}/\mathrm{cm}^3$ is many times larger than the intrinsic carrier density, so the carrier density we need in the calculation of the conductivity ($\sigma = qn\mu$) is approximately the donor density ($n = N_D$). Next we must assume a reasonable value for the mobility of donor electrons in Ge. Let us take as an order-of-magnitude approximation $10^3\ \mathrm{cm}^2/\mathrm{V}\cdot\mathrm{s}$. Then

$$\sigma = \left(1.60 \times 10^{-19}\ \mathrm{C}\right)\left(10^{14}\ /\mathrm{cm}^3\right)\left(10^3\ \mathrm{cm}^2/\mathrm{V}\cdot\mathrm{s}\right) = \frac{0.016\ \mathrm{C}}{\mathrm{V}\cdot\mathrm{s}\cdot\mathrm{cm}} = 1.6/\quad \cdot\mathrm{m}. \tag{9}$$

Using a relationship between the hole and electron densities and the intrinsic carrier density ($np = n_i^2$) and the approximation $n \approx N_D$ led to eq. (1.12) in *Melissinos*, which

yields in this particular case

$$p_n \approx \frac{n_i^2}{N_D} \approx 2.3 \times 10^{12}/\mathrm{cm}^3. \tag{10}$$

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4. *Melissinos* exercise 1.4.

Make a plot of the Fermi-Dirac distribution at $T = -78^\circ\mathrm{C}$, room temperature, and a $T = 500^\circ\mathrm{C}$ when $E_F = 1\ \mathrm{eV}$.

(Accurate plots are expected. First plot for $0 < E < 2\ \mathrm{eV}$ *and then for* $0.8\ \mathrm{eV} < E < 1.2\ \mathrm{eV}$*.)*

The Fermi-Dirac distribution is given by

$$f(E) = \frac{1}{\exp[(E - E_F)/kT] + 1}, \tag{11}$$

where E_F is the Fermi energy. Let us express kT for the various temperatures in eV's:

T (in C)	kT in eV's
-78° C	$0.017\ \mathrm{eV}$
17° C	$0.025\ \mathrm{eV}$
500° C	$0.067\ \mathrm{eV}$

Now let us plot the Fermi distribution for the range $0 < E < 2$

$1/(\exp((E - 1.0)/0.017) + 1)$

and then in a smaller region surrounding the Fermi energy ($0.8 < E < 1.2$)

This function gives the average number of fermions filling a level with energy E. Because we are talking about fermions, $f(E)$ varies between zero and one. At $T = 0$, $f(E)$ is a step function — equal to one for $E < E_F$ and equal to zero for $E > E_F$.