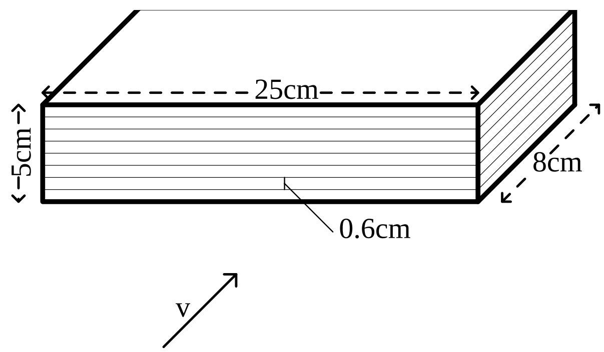
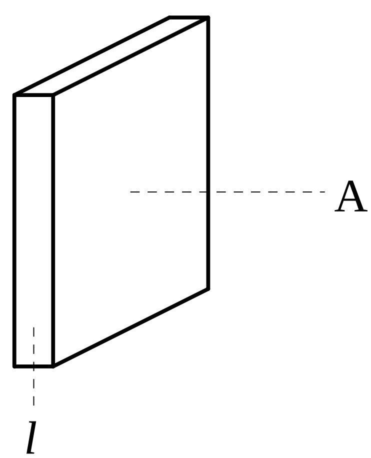
1. In a desktop electrostatic air cleaner, air is driven through the "cell" by a fan that draws 55 W (when set at "high"). The cross sectional dimensions of the cell are 25 cm by 5 cm. Neglecting losses, you can compute the speed of air flow, v, from these data. Compare this computed value of v with the value "measured" in class. (At the calculated v) how many minutes does it take to process a volume of air equal to that of a typical room? (Take the room to be 4 by 4 by 2 cubic meters).



After an initial period when the cleaner is first turned on, a steady stream of air is passing through it at a constant velocity. (To envision the "steady state" situation, it might help to think of the system as a long tube of length L and cross sectional area A that forms a closed circuit, but it's not necessary.) The air would come to a standstill rather quickly if we were not continually supplying it with energy, and the energy we must supply to the air is its (mechanical) kinetic energy (as opposed to its thermal kinetic energy). A volume of gas with dimensions ℓ by A has mass $m = \rho \ell A$, where ρ is the mass density of air.



We must supply to that volume of air an energy $\Delta E = \frac{1}{2}mv^2$, and the time in which we have to supply that energy is the time it takes for that volume to pass by the fan which is $\Delta t = \ell/v$. Therefore, the power, which is energy per time, is

$$P = \frac{\Delta E}{\Delta t} = \frac{\frac{1}{2}\rho \ell A v^2}{\ell/v} = \frac{\rho A v^3}{2}. \tag{1}$$

Solving for v yields

$$v = \left(\frac{2P}{\rho A}\right)^{1/3} \tag{2}$$

(Note that ℓ dropped out so that it could have been taken to be L, the imagined length of the entire system, or taken to be infinitesimally small.) Using that $\rho_{\text{air}} = 1.293$ kg/m^3 (from *Tipler*, p. 333) and the information above, we find that $v \approx 19$ m/s.

This velocity is somewhat high. It's equivalent to about 43 miles/hour *which would be a very high wind. So our velocity is probably* 2 *to* 3 *times too high. Thus not all of the power is going into translational kinetic energy. Where is it going?*

The product Av (which equals 0.24 m^3/s) provides the volume of air passing by the fan per unit time. From it we can calculate the time needed to process a volume of 32 m^3 to be 130 s or 2.2 min at this speed.

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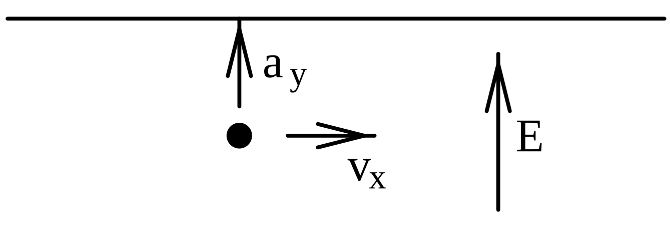
2. Grains of pollen of mass M are entrained (carried along) by air moving at speed v, as calculated in problem 1, between plates held at a potential difference $\mathcal{V} = 5500$ Volts. The distance d between plates is 0.6 cm and the depth of a cell is 8 cm. Will the pollen be collected for sure on the positive plate when a grain picks up the elementary charge e or does it have to be multiply charged? I am particularly concerned with oak pollen that has very small grains, only 10 micrometers in diameter (you can estimate M from this or you can find out what M really is).

If we assume the pollen grains to have a density the same order of magnitude as water ($\rho_{\text{water}} = 1 \times 10^3$ kg/m^3) and a spherical shape, then we can calculate the mass M from the diameter given above

$$M = \frac{4\pi}{3}\left(\frac{D}{2}\right)^3 \rho_{\text{water}}, \tag{3}$$

which gives $M \approx 5.2 \times 10^{-13}$ kg. (*The factor* $4\pi/3$ *is overkill in such an order of magnitude calculation.*)



Next we can calculate the acceleration due to the electric field the particle is passing through. The electric field between the plates is $E = \mathcal{V}/d$, where $\mathcal{V}$ is the voltage difference between plates and d the distance between plates. (We are neglecting edge effects.) Multiplying by the charge e gives the force, and dividing by M the acceleration

$$a_y = \frac{e\mathcal{V}}{Md}, \tag{4}$$

which yields $a_y = 0.28$ m/s^2. (*Note that we have neglected friction in this calculation as well as any turbulent motion of the air as it passes through the air cleaner.*)

Now we find the time it takes to pass through the plates as $t = s_x/v_x$ where v_x was calculated in the previous problem. We obtain $t = 4.2 \times 10^{-3}$ s. And finally we determine the deflection in the y direction from $s_y = \frac{1}{2}a_y t^2$ giving $s_y = 2.5 \times 10^{-6}$ m.

So a singly charged particle moves a couple micrometers, falling far short of the 0.6 cm it requires to ensure that it would hit the plate before leaving the air cleaner. We have suggested above that our calculation of v was too high, and this might change s_y by a factor of 10, but we need it to change by a factor closer to 10^3 to ensure trapping. Thus if particles of mass M estimated above are to be trapped, they must be multiply charged, with charges 100 to 1000 times that of a single electron.

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3. Sparks and discharges (as seen in "Jacob's ladder") emit bluish light. What are the frequency and wavelength of this light? What is the energy of an emitted photon in eV? You can give a range of values or a typical value.

Blue light has a wavelength ranging roughly from 4.8×10^{-7} m to 4.2×10^{-7} m. The corresponding frequency range is 6.3×10^{14} Hz to 7.1×10^{14} Hz ($f = c/\lambda$).

The energy of a photon is given by $E = hf = \hbar\omega$ (*Tipler*, p. 1148) where h is Planck's constant ($h = 4.136 \times 10^{-15}$ eV $\cdot$ s). Thus the associated energy range is 2.6 eV to 2.9 eV.

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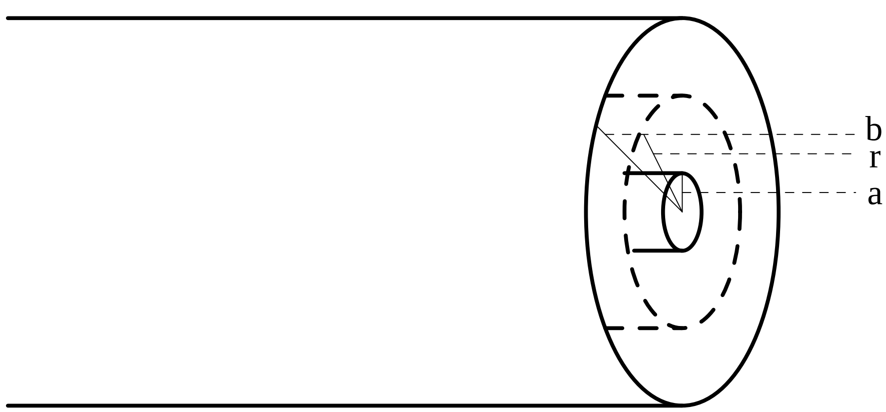
4. A steel wire of radius a hangs at the center of a steel can of radius b. If the wire is at potential V and the pipe is grounded, what is the field at the surface of the wire? I am asking for an exact answer in the idealized case of wire and can of infinite length. Assume that the radius of the can is 5 cm, what practically feasible a and V will give a good corona discharge? You can just give typical values.



Applying Gauss' law

$$\oint \mathbf{E} \cdot d\mathbf{A} = \frac{Q_{\text{inside}}}{\varepsilon_0} \tag{5}$$

to the cylindrical geometry, we find

$$E(r)\ 2\pi r L = \frac{Q_{\rm inside}}{\varepsilon_0}, \tag{6}$$

where r is the radius of the enclosing surface (with $a \le r \le b$) and L is its length. (In the infinitely long case, the electric field points radially, so there is no contribution to the integral above from the end caps of the cylindrical surface.) Solving for $E(r)$ gives

$$E(r) = \frac{\lambda}{2\pi\varepsilon_0 r}, \tag{7}$$

where $\lambda = Q_{\rm inside}/L$, the linear charge density of the wire.

But we do not know the linear charge density; we know the applied voltage. We can calculate the potential difference from the electric field through

$$\begin{aligned} V_b - V_a &= -\int_a^b \mathbf{E}(\mathbf{r}) \cdot d\mathbf{r} \\ &= -\frac{\lambda \ln(b/a)}{2\pi\varepsilon_0}. \end{aligned}$$

Hence, the linear charge density is

$$\lambda = -\frac{2\pi\varepsilon_0 \mathcal{V}}{\ln(b/a)}, \tag{8}$$

where $\mathcal{V} = V_b - V_a$. Substituting this expression for λ into the electric field gives

$$E(r) = -\frac{\mathcal{V}}{\ln(b/a)\ r}. \tag{9}$$

The electric field at the surface of the wire is $E(a) = \mathcal{V}/\ln(b/a)a$.

Dielectric breakdown in air occurs at an electric field of roughly 3×10^6 V/m (*Tipler*, p. 677). It occurs when the neutral molecules become ionized, then there are free charges about to conduct electricity. Some ions are around before breakdown, but at breakdown these ions gain enough energy as they are accelerated by the electric field that when they collide with a neutral atom they cause it to ionize as well—leading to a "chain reaction."

We don't want breakdown to occur over a wide region of our system (because that will cause a spark). We only want breakdown over a small region surrounding the wire, which will serve as a source of ions that spray into the rest space but don't necessarily cause further ionization.

Our first constraint is that the field at $r = a$ must be greater than the breakdown field, i.e.

$$\frac{\mathcal{V}}{\ln(b/a)a} > 3 \times 10^6 \text{ V/m}. \tag{10}$$

The factor $\ln(b/a)$ won't ever be very large. If the ratio $b/a = 100$ then $\ln(b/a) \approx 4.6$; and if $b/a = 1000$ then $\ln(b/a) \approx 6.9$. The important factor is the ratio $\mathcal{V}/a$. If the wire has a radius of 0.5 mm, then $\mathcal{V} > 6900$ V. Let us assume $\mathcal{V} = 10,000$ V, then at $r \approx 0.72$ mm, the field falls below the breakdown threshold. So indeed the dielectric breakdown is only occurring in a small region surrounding the wire.

If there is hot, polluted air rising through the cylinder like the one in this problem, it can be used as a "precipitator." See *Tipler*, p. 682-684 for a short description of an electrostatic precipitator. That section also includes a description of xerography. See also Bloomfield p. 378-383.