Assignment 7

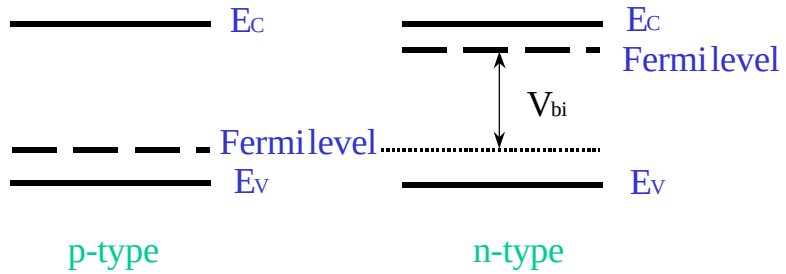
Problem 3

3. In a p-n junction, what causes the built-in bias V_{bi}? In eq. $(1.18'')$ of *Melissinos*, what are precisely k, T, q, N_D, N_A and n_i?

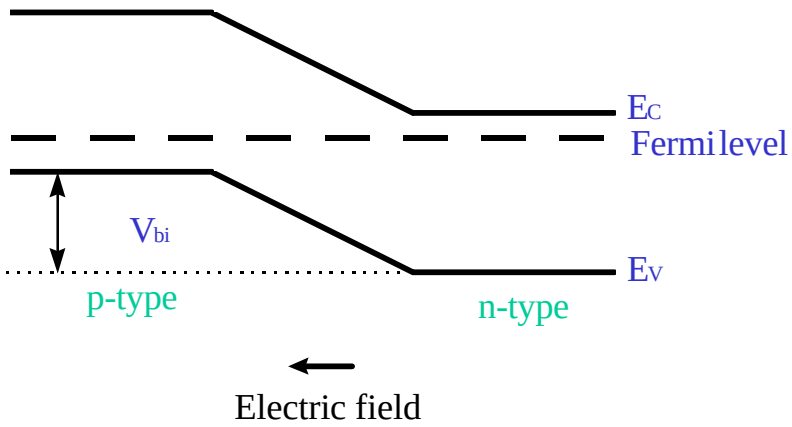
Some of these quantities are fundamental constants. What are the values of the others for a typical commercial junction? Check that they are consistent with the value $V_{bi} = 0.65\,\mathrm{eV}$ quoted by *Melissinos*.

Let us rederive eq. $(1.18'')$ in a manner different from *Melissinos* and identify the various quantities as they arise. As discussed in problem 1 of assignment 5, when an n-type material and a p-type material are brought in contact, electrons diffuse from the former to the latter. In doing so, the electrons can lower their energy and also reduce the concentration gradient. In a somewhat more sophisticated approach, we know that the difference in "chemical potentials" is what drives the migration of particles. Recall that E_F plays the role of the chemical potential for these systems. The band structure before and after the junction formation are shown below.

Before



After



The two semiconductors, which were originally neutral, are now charged (at least in

the region immediately surrounding the junction): the p-type becomes negatively charged as the electrons from the n-type come over. This charge separation costs energy, and that is what eventually stops the current from flowing. In the more sophisticated language, migration stops when the chemical potentials becomes equal.

There is an electric potential (V_{bi}) associated with the separated charges, and it is this potential that compensates for the initial difference in chemical potential. So if we find that initial difference in chemical potential, we find V_{bi}.

Recall *Melissinos*, eqs. (1.8)

$$\begin{aligned} n &= N_c \exp[-(E_c - E_F)/kT] \\ p &= N_v \exp[-(E_F - E_v)/kT], \end{aligned} \tag{1}$$

where

$$\begin{aligned} n &- \text{electron carrier density} \\ p &- \text{hole carrier density} \\ N_c &- \text{density of states in conduction band} \\ N_v &- \text{density of states in valence band} \\ E_c &- \text{lowest energy in the conduction band} \\ E_v &- \text{highest energy in the valence band} \\ E_F &- \text{the Fermi level (the chemical potential)} \end{aligned} \tag{2}$$

and where k is the Boltzmann constant (1.381×10^{-23} J/K) and T the temperature in Kelvin.

The formula above holds for both doped and undoped cases. N_c, N_v, E_c and E_v essentially do not change upon doping; n, p and E_F do. Let us use the subscript n to refer to n-type doping and p to refer to p-type doping. In the n-type material, the electron carrier density n_n is approximately equal to N_D, the density of ionized donors; and in the p-type material, the hole carrier density p_p equals N_A, the density of ionized acceptors. Then we have

$$\begin{aligned} N_D &= N_c \exp[-(E_c - E_{Fn})/kT] \\ N_A &= N_v \exp[-(E_{Fp} - E_v)/kT]. \end{aligned} \tag{3}$$

and it follows that

$$N_D N_A = N_c N_v \exp[-(E_c - E_v)/kT] \exp[(E_{Fn} - E_{Fp})/kT]$$

As shown by Melissinos on page 11, the intrinsic carrier density n_i is given by

$$n_i^2 = N_c N_v \exp[-(E_c - E_v)/kT]$$

Hence we find

$$N_D N_A = n_i^2 \exp[(E_{Fn} - E_{Fp})/kT]$$

which can be rewritten as

$$E_{Fn} - E_{Fp} = kT \ \ln\left(\frac{N_D N_A}{n_i^2}\right). \tag{4}$$

This expression is for the initial difference in chemical potentials between the n-type and

p-type materials, which we argued above was equal to the potential caused by the charge separation. In *Melissinos'* formula V_{bi} is an electric potential, so we must divide the potential above by a charge — in this case the elementary charge (q in *Melissinos'* notation), so that

$$V_{bi} = \frac{kT}{q} \ln\left(\frac{N_D N_A}{n_i^2}\right), \tag{5}$$

The quantities above were identified along the way, but let us recap

$$\begin{aligned}
V_{bi} &- \text{“built}-\text{in}'' \text{ voltage difference} \\
k &- \text{Boltzmann constant } (8.621 \times 10^{-5} \text{ eV}) \\
T &- \text{temperature in Kelvin} \\
q &- \text{elementary charge } (1.6 \times 10^{-19} \text{ C}) \\
N_D &- \text{density of ionized donors in the n} - \text{type material} \\
N_A &- \text{density of ionized acceptors in the p} - \text{type material} \\
n_i &- \text{the intrinsic (undoped) carrier density of both electrons and holes.}
\end{aligned} \tag{6}$$

Melissinos gives $n_i = 10^{10}/\text{cm}^3$ as the intrinsic carrier density of silicon at room temperature (where $kT = \text{eV}/40$). N_D and N_A correspond to the doping density. In the solutions to problem 2 of assignment 5, we saw the number density of Si is $5.00 \times 10^{22}\ /\text{cm}^3$. So if we doped at 1 part in a billion, it would correspond to a doping density of $5.00 \times 10^{13}\ /\text{cm}^3$. That would correspond to a rather light doping, so let us take $N_D = N_A = 10^{15}/\text{cm}^3$. That would give

$$V_{bi} = \frac{\text{eV}/40}{q} \ln\left(\frac{(10^{15}\ /\text{cm}^3)^2}{(10^{10}\ /\text{cm}^3)^2}\right) \approx 0.576\ V. \tag{7}$$

Our estimate of the doping density was a bit low, a density of approximately $4.4 \times 10^{15}/\text{cm}^3$ would give $V_{bi} = 0.65$ V.