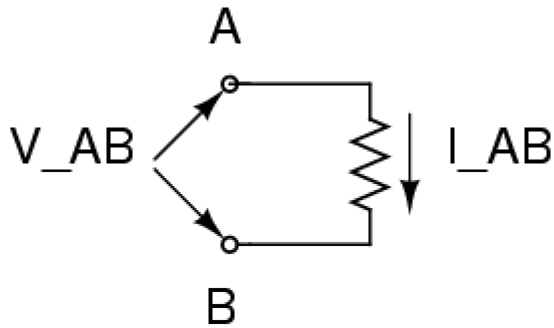
The Resistor:



V_AB ≡ Voltage of point A w.r.t. Point B

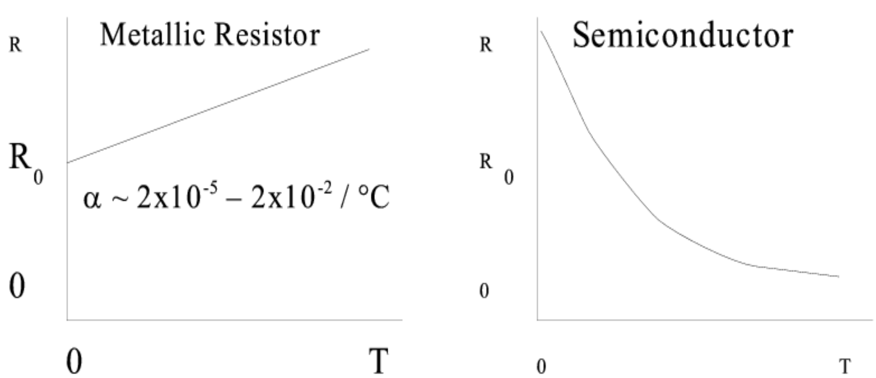
I_AB ≡ current flowing from A to B

Satisfies Ohm's Law:

V_AB = I_AB * R (R is called ***resistance***)
or
I_AB = V_AB * G (G is called ***conductance***)

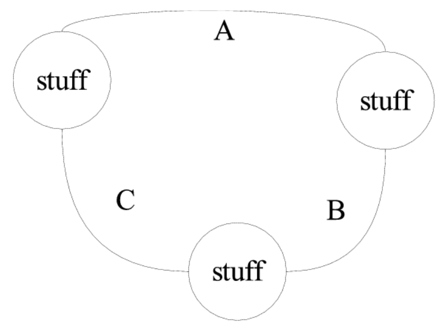
note: 1) obviously G = 1/R
2) if V_AB > 0, then a current flows from A to B (for a positive resistance)

(Units: R in [Ohms], I in [Amps], V in [Volts], G in [Siemens] formerly [Mhos])



Metallic conductors: electron charge carriers, more collisions as temperature increases, relative linearity of effect useful for T measurements.
Semiconductors: more electron/hole pairs available as temperature increases

Kirchoff's Laws:



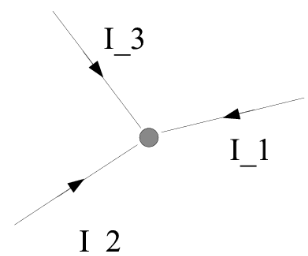
Kirchoff's Voltage Law (KVL): Potentials around a closed loop add to zero.

In general:

$$\sum_{loop} \Delta V = 0$$

$$V_{AB} + V_{BC} + V_{CA} = 0$$

This is a statement of conservation of energy: No net energy used to transport a unit charge around a closed loop.



Kirchoff's Current Law (KCL): Conservation of charge. Charge cannot build up indefinitely at a node.

In general:

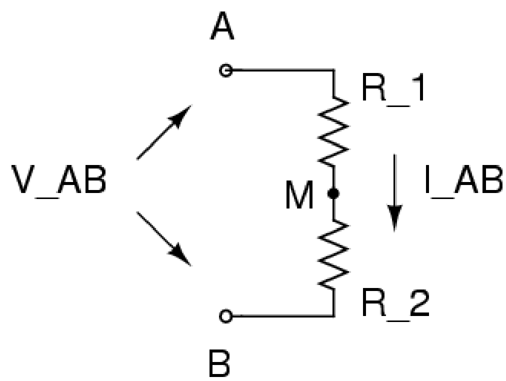
$$\sum_{into\,node} I = 0$$

$$I_1 + I_2 + I_3 = 0$$

Using Ohm's and Kirchoff's Laws for solving circuit problems:

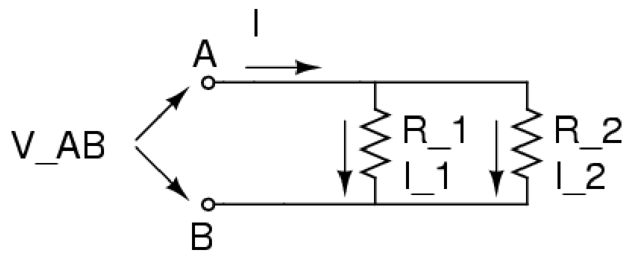
Resistor Combinations

Series combination



KVL: V_AB = V_AM + V_MB = -V_BA
Ohm: V_AM = I * R_1, V_MB = I * R_2
V_AB = I * (R_1 + R_2)
therefore: R_total = R1+R2

Parallel combination



KCL: I = I_1 + I_2
Ohm: I = G_total * V_AB
= (G_1 + G_2) * V_AB
therefore: G_total = G_1+G_2
or: 1/R_total = 1/R_1 + 1/R_2

General comments:

$R_1 \| R_2 = \frac{R_1 R_2}{R_1 + R_2}$ but useful limiting cases are: $(i)\, R_1 = R_2 \rightarrow R_1 \| R_2 = \frac{R_1}{2} = \frac{R_2}{2}$

$(ii)\, R_1 \gg R_2 \rightarrow R_1 \| R_2 \simeq R_2$

In practice 10x difference in resistor values is big enough to give a ~10% approximation to (ii).

For N resistors combined:

series $R = R_1 + R_2 + R_3 + \ldots R_N$

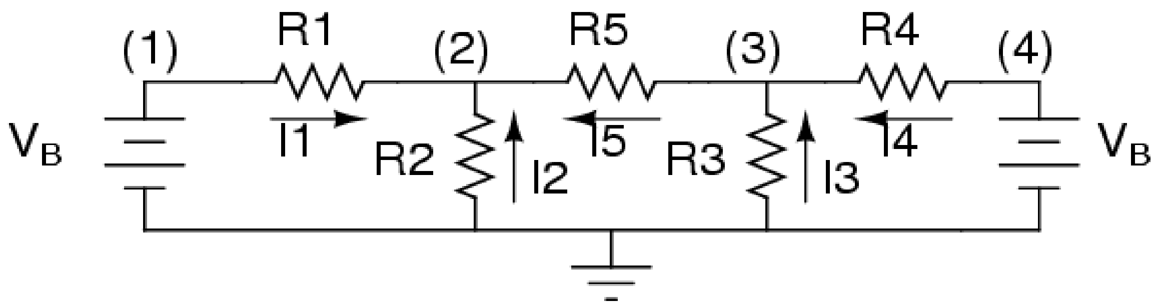
parallel $\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \ldots + \frac{1}{R_N}$

Network Analysis

Only the most simple circuits can be solved by reducing connections to parallel and series combinations of circuits. In general solving for the voltages and currents in a circuit composed of resistors will lead to a set of linear equations solvable by methods of linear algebra.

One technique for analyzing linear networks is the "Nodal Method." (Used by SPICE, a computer program for simulating electronic circuits). The Nodal Method solves for voltages at the nodes by applying KCL at each node.

The technique is applied to the circuit below. V_A and V_B are given voltages, the lower symbol in the schematic represents "Ground" or "Common" --- the defined 0 voltage point of the circuit.



Nodes (1) and (2) are simple: $V_1 = V_A \quad V_4 = V_B$
For Node 2:
KCL: $I_1 + I_2 + I_5 = 0 \rightarrow G_1(V_1 - V_2) + G_2(0 - V_2) + G_5(V_3 - V_2) = 0$

$$G_1 V_1 - (G_1 + G_2 + G_5) V_2 + G_5 V_3 = 0$$

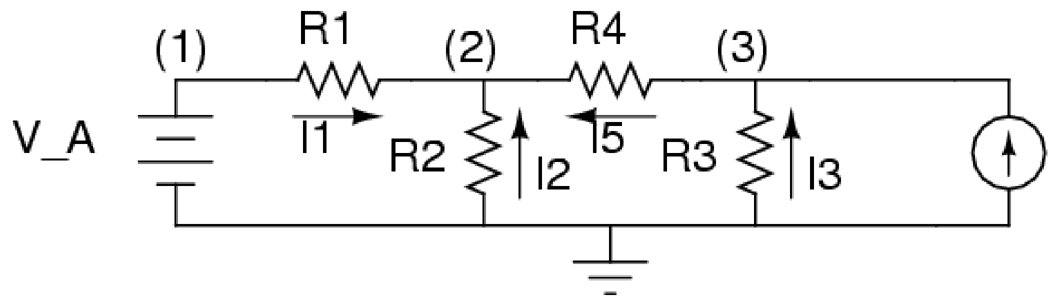
For Node 3:

$$KCL: I_3 + I_4 - I_5 = 0 \rightarrow G_3(0 - V_3) + G_4(V_4 - V_3) - G_5(V_3 - V_2) = 0$$

$$G_5 V_2 - (G_3 + G_4 + G_5) V_3 + G_4 V_4 = 0$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ G_1 & -(G_1+G_2+G_5) & G_5 & 0 \\ 0 & G_5 & -(G_3+G_4+G_5) & G_4 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \end{bmatrix} = \begin{bmatrix} V_A \\ 0 \\ 0 \\ V_B \end{bmatrix}$$

This method is not restricted to only voltage sources. Current sources can also be added. The network below produces the following set of linear equations:



$$\begin{bmatrix} 1 & 0 & 0 \\ G_1 & -(G_1+G_2+G_4) & G_4 \\ 0 & -G_4 & G_3+G_4 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} V_A \\ 0 \\ I_B \end{bmatrix}$$

Notes:

i) To get currents use I1 = G1(V1-V2), if V1-V2 < 0 then the current flows in the opposite direction of the arrow.

ii) These examples used ideal voltage and current sources
  - For an ideal voltage source Vout is independent of the amount of current drawn from the source
  - For an ideal current source Iout is independent of the voltage across the sources

All circuits consisting of only resistors and sources will produce linear equations such as those above. Therefore the **Superposition Theorem** holds:

> The total voltage at a node or the total current through a branch of the circuit equals the sum of voltages or current due to each source separately. i.e. Short across all but one voltage source or leave open circuit all but one current source.

For example: The voltage at point 'M' in the following circuit and the current through R4 may be calculated as follows:

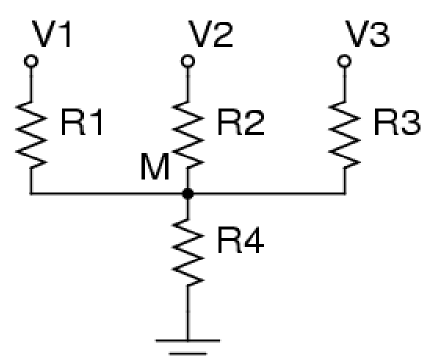
i) set V1=V2=0, calculate Voltage at M and current through R4 w.r.t. voltage source V3 (V003, I003)

ii) set V2=0=V3, calculate V/I (V100, I100)

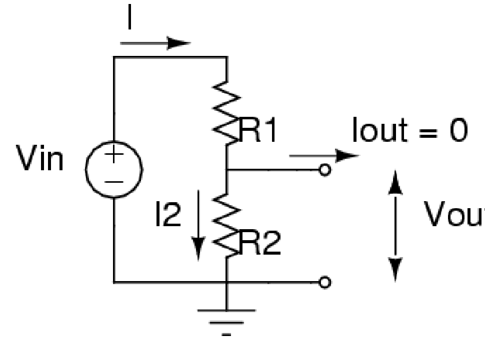
iii) set V1=0=V3, calculate V/I (V020, I020)

iv) Voltage at M = V100+V020+V003

v) Current through R4 = I100+I020+I003



Voltage Divider



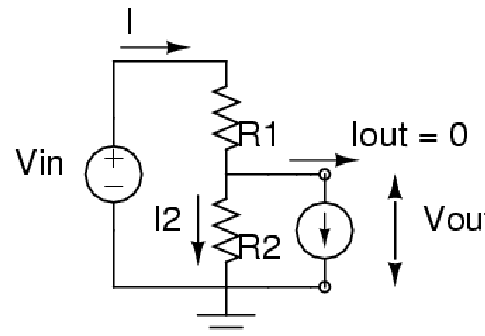
This is a common circuit fragment that will appear many times in the analysis of more complex circuits.

First suppose no current flows to the output, i.e. I2=I then

$$V_{out} = I_2 R_2 = I R_2$$

using $I = \frac{V_{in}}{R_1 + R_2}$

gives $V_{out} = \frac{R_2}{R_1 + R_2} V_{in}$



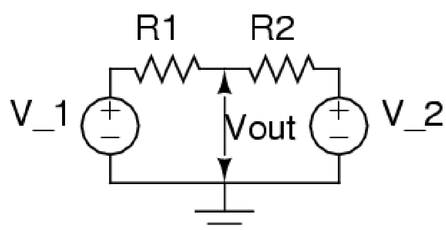
This is still approximately true as long as I_out is sufficiently small, more precisely:

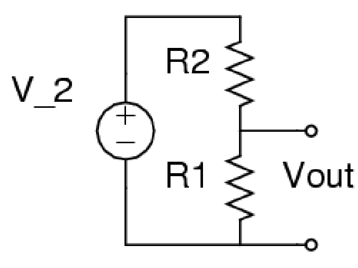
$$I_2 = I_1 - I_{out} \rightarrow V_{out} G_2 = (V_{in} - V_{out}) G_1 - I_{out}$$

$$\rightarrow V_{out} = V_{in} \frac{G_1}{G_1 + G_2} - I_{out} \frac{1}{G_1 + G_2}$$

$$= V_{in} \frac{R_2}{R_1 + R_2} - I_{out} \frac{R_1 R_2}{R_1 + R_2}$$

Voltage Combiner/Adder





V_1 set to 0

Set either V_1 or V_2 to 0 and use superposition theorem

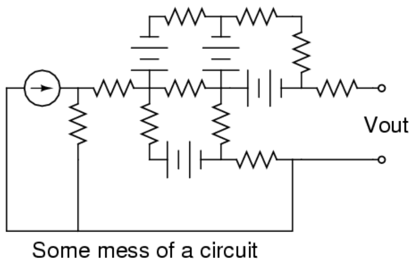
1) $V_1 = 0$: circuit becomes a voltage divider (above right) $V_{out} = V_2 \frac{R_1}{R_1 + R_2}$

2) $V_2 = 2$: $V_{out} = V_1 \frac{R_2}{R_1 + R_2}$

$\rightarrow$ by superposition $V_{out} = V_1 \frac{R_2}{R_1 + R_2} + V_2 \frac{R_1}{R_1 + R_2}$

Thevenin's Theorem

When analyzing a complicated circuit is is useful to break it down into "subcircuits". One complication that may arise is that the input current required by a later subcircuit may affect the output voltage of the preceding subcircuit. Thevenin's Theorem provides a way for representing the output of any linear circuit in terms of an ideal voltage source in series with a resistor.



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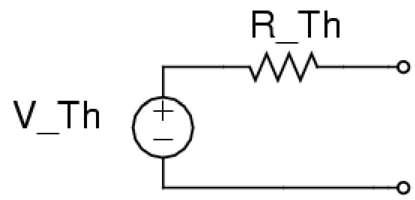
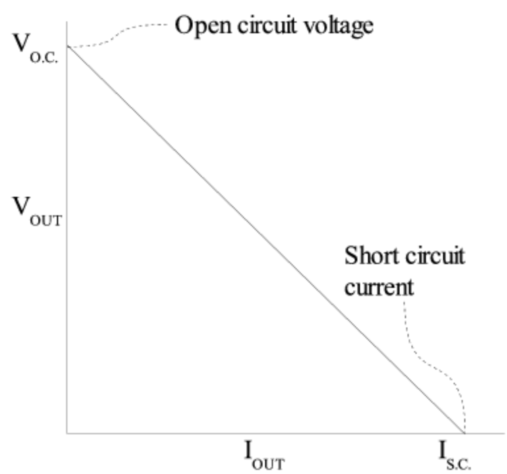


1st note that in a linear circuit the voltages and currents are linearly dependent; specifically V_out vs. I_out is a straight line.

This V_out versus I_out plot is sometimes called a "load line." The equation of this line is given by:

$$V_{out} = V_{Th} - I_{out} R_{Th}$$

The load line of the Thevenin equivalent circuit will be identical to our circuit in question if:



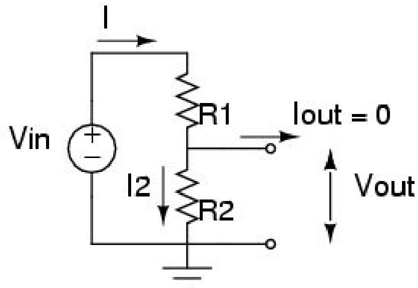
$V_{Th} = V_{O.C.}$ open circuit voltage

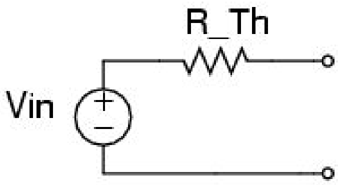
$R_{Th} = \frac{V_{O.C.}}{I_{S.C.}}$ where $I_{S.C.}$ is the short circuit current

Voltage divider revisited

$$V_{OC}=V_{in}\frac{R_1}{R_1+R_2}=V_{Th}$$

$$I_{SC}=\frac{V_{in}}{R_1} \qquad \rightarrow R_{Th}=\frac{V_{OC}}{I_{SC}}=\frac{R_1 R_2}{R_1+R_2}$$





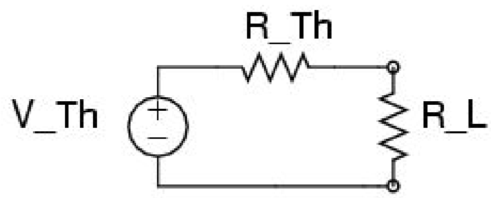
$$V_{out}=V_{in}\frac{R_2}{R_1+R_2}-I_{out}\frac{R_1 R_2}{R_1+R_2}$$

Compare to the first voltage divider solution.

In practice one doesn't actually short the output of a circuit to find R_Th or its "output impediance". Instead you load the circuit with a resistance small emough to cause the circuits output to drop appreciably from its unloaded value.

1) When unloaded R_L = ∞
   V_out = V_OC = V_TH

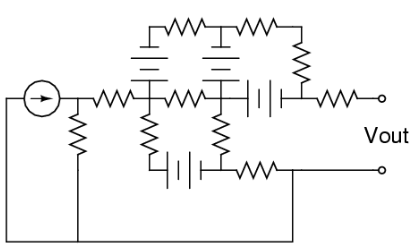
2) With finite R_L one has a voltage divider



$$V_{out}(R_L)=V_{Th}\frac{R_L}{R_{Th}+R_L}$$

$$\rightarrow R_{Th}=R_L\left(\frac{V_{Th}}{V_{out}}-1\right)$$

<u>Norton's Theorem</u> – Dual to Thevenin's Theorem



Some mess of a circuit

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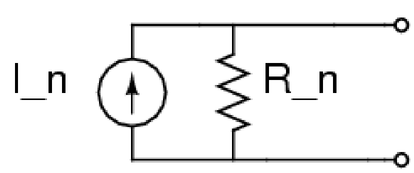
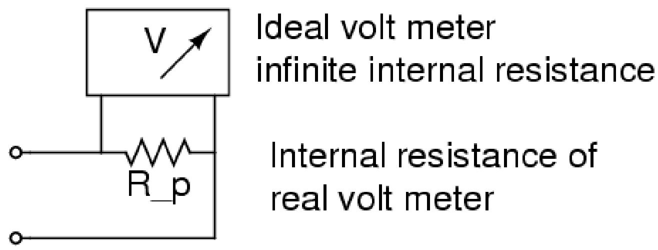


where $I_n=I_{SC}$ $\qquad R_n=V_{OC}/I_{SC}$

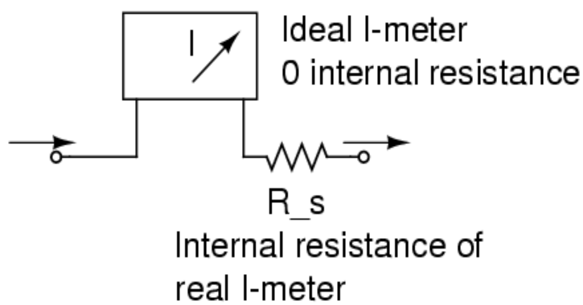
Measuring Instruments (and their limitations)

An ideal voltmeter would measure the voltage between two points of a circuit without requireing any input current from the circuit in order to perform the measurement. In reality a voltmeter requires a small but finite input current.

A representation of a real voltmeter is a parallel combination of an ideal voltmeter with a resistor.



An ideal current meter would be inserted into a branch of a circuit to measure current flow without causing any additional voltage drop. i.e. It would have 0 Ohms input resistance.



A real I-meter will cause a small voltage drop when measureing current. Our model will include an internal series resistor.

Power in DC circuits

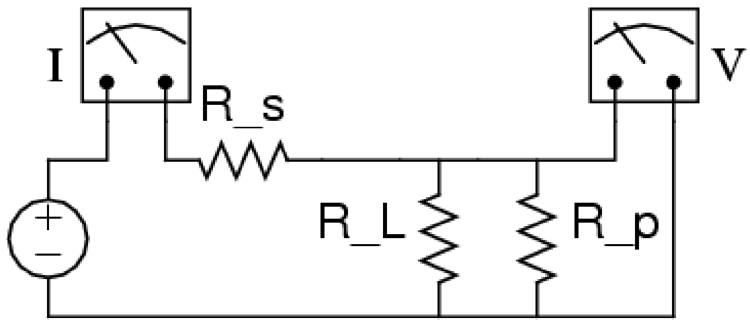
The power dissapated by an element in a DC circuit is given by:

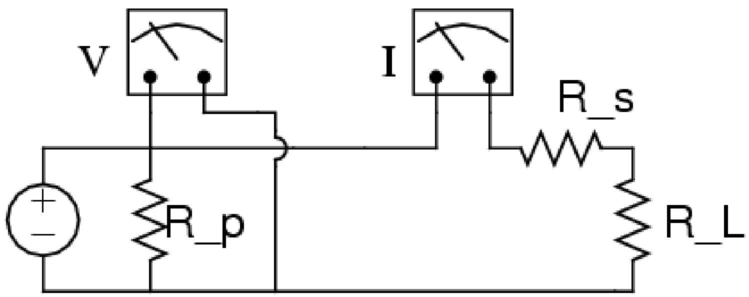
$P = VI$, where V is the voltage across an element and I is the current flowing through the element.

For a resistive circuit,an equivalent form is: $P = I^2R$

When measuring V across a load and I through a load there are two possible configurations.

V measured across load, but I measures current through R_L and R_p. Therefore this configuration should be used when R_L<<R_p.





I measured through load, but V measured across both load and I-meter. Therefore this configuration should be used when R_L>>R_s

If neither of these conditions can be fullfilled the one must correct the measurement readings for the effects of the meters.

Appendix

Reading resistors via color codes

Carbon resistors used in the lab are often marked with color codes to show the value of the resistor in units of Ohms. The resistor will typically have three to four color bands. The first two bands give the value of the resistor to two significant figures, the third band gives the multiplier value used to define the resistance. A forth band is used to designate the tolerance or accuracy of the nominal value. Gold = 5%, Silver = 10%, no marking implies a 20% tolerance.

For example a resistor marked with:
Brown – Black – Red stripes is 1000 Ohms.
Yellow – Violet – Brown stripes is 470 Ohms.

Basic resistor color code		
color	**Numerical digit**	**Multiplier value**
BLACK	0	1
BROWN	1	10
RED	2	100
ORANGE	3	1000
YELLOW	4	10000
GREEN	5	100000
BLUE	6	1000000
VIOLET	7	
GREY	8	
WHITE	9	