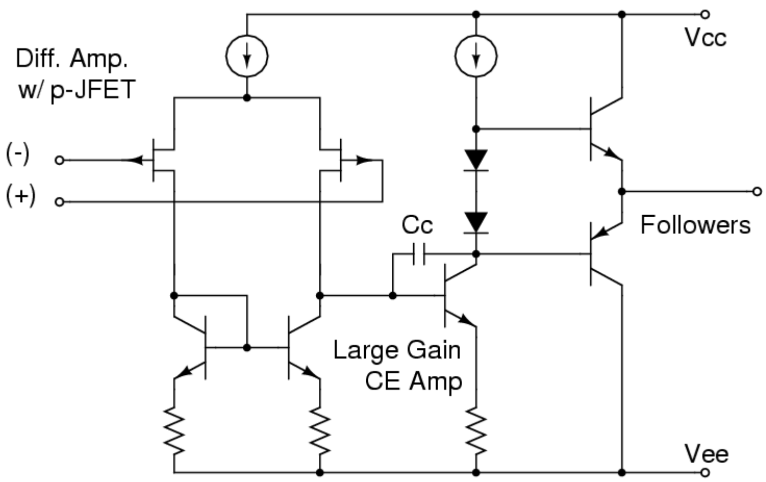
OPAMPs I: The Ideal Case

The basic composition of an operational amplifier (OPAMP) includes a high gain differential amplifier, followed by a second high gain amplifier, followed by a unity gain, low impedance, output stage. A simplified model for the LF411 used in class is shown below:

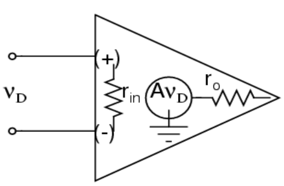


(+) = non-inverting input
(-) = inverting input

V_{out} has the same sign as $V_{(+)} - V_{(-)}$

(the capacitor limits the gain at high frequency, more on this later)

To do calculations, we will use a small signal equivalent circuit for the OPAMP[1].



Universal assumptions for OPAMPs:

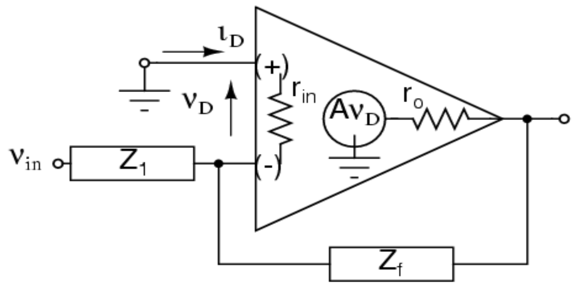
1. Unloaded $v_o = A v_D$ where $v_D = v_{(+)} - v_{(-)}$
   A is large, typically 10^5
2. r_{in} is large $\rightarrow \infty$
3. r_o is small $\rightarrow 0$

Note: there are no explicit resistors from inputs to ground or power supplies. This is consistent w/ ignoring input bias currents and having a current source in the Diff. Amp.

1The small signal analysis will avoid imperfections of a D.C. nature, such as input bias current, input voltage offset, and bias current offset. We'll treat these later.

Negative feedback: Assume that $v_D = v_{(+)} - v_{(-)} \neq 0$. The idea is to take a fraction B of v_o and add it back into the input in order to cancel v_D. There are two topologies to consider:

Voltage Shunt feedback

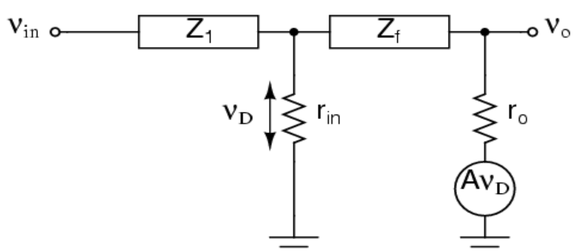


We will derive the "Golden Rules" from our above assumptions

1. $v_D \approx 0$
2. inputs draw no current, $\iota_D \simeq 0$
3. $Z_{out} \simeq 0$

Proof of assertion (1) : $v_D \approx 0$

Draw the input-output circuit:



By superposition:

$$v_D = v_{in} \frac{r_{in} \| (Z_f + r_o)}{Z_1 + [r_{in} \| (Z_f + r_o)]} + A v_D \frac{(r_{in} \| Z_1)}{(r_o + Z_f) + (r_{in} \| Z_1)}$$

Use the approximations: $r_{in} \gg |Z_1|, |Z_f|$ and $r_o \ll |Z_f|$

$$v_D \simeq v_{in} \frac{Z_f}{Z_1 + Z_f} + A v_D \frac{Z_1}{Z_f + Z_1}$$

$$\Rightarrow v_D = \frac{-v_{in}(1 - B)}{BA - 1} \quad \text{where} \quad B = \frac{Z_1}{Z_1 + Z_f} \quad \text{is called the feedback ratio}$$

and $v_D \to 0$ as long as $BA >> 1$

Proof of assertion (2): no current into inputs follows from (1) since $\iota_D = \frac{v_D}{r_{in}} \to 0$

Proof of assertion (3): $Z_{\text{out}} \simeq 0$ Use Thevenin's theorem

open circuit: $v_{\text{o}}^{\text{o.c.}} = A v_D = -v_{\text{in}} \dfrac{A(1-B)}{BA-1}$

short circuit: $\iota_{\text{o}}^{\text{s.c.}} = \dfrac{1}{r_{\text{o}}} A v_D^{\text{s.c.}}$ for output shorted to ground $v_D = -(1-B) v_{\text{in}}$

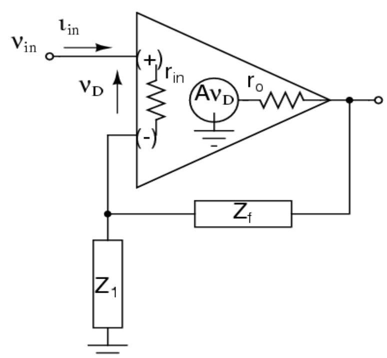
$\Rightarrow Z_{\text{out}} = \dfrac{v_o^{\text{o.c.}}}{\iota_o^{\text{s.c.}}} = \dfrac{r_{\text{o}}}{BA-1}$ therefore $Z_{\text{out}} \simeq 0$ for $BA \gg 1$

As a corollary to assertion (1) it is easy to see that the gain for the entire circuit, including feedback (the "closed loop gain") is:

$$G = \frac{v_{\text{o}}}{v_{\text{in}}} = \frac{A v_d}{v_{\text{in}}} = \frac{-A(1-B)}{BA-1} \rightarrow -\frac{Z_f}{Z_1} \quad \text{for} \quad BA \gg 1$$

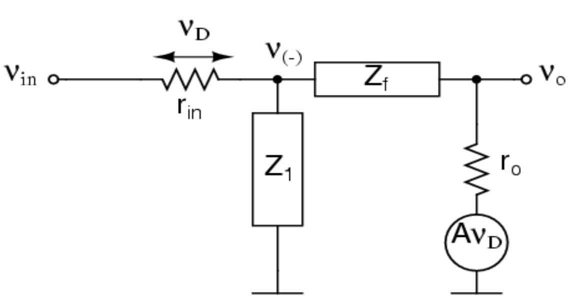
Notice that G is essentially independent of the details of A, the open loop gain. Even if *A* is non-linear for large signals, e.g. "Barn roof distortion", G is still constant. All that matters is that *BA*>>1. Through use of negative feedback we are "trading" extremely large gain *A* for moderate gain $\sim Z_f/Z_{in}$ with excellent linearity.

Voltage Series feedback



We will again derive the "Golden Rules"

1. $v_D \approx 0$
2. inputs draw no current, $\iota_D \simeq 0$
3. $Z_{\text{out}} \simeq 0$



Again proceeding by superposition:

$$v_{(\text{-})} = v_{\text{in}} - v_D = v_{\text{in}} \frac{Z_1 \| (Z_f + r_o)}{r_{\text{in}} + [Z_1 \| (Z_f + r_o)]} + A v_D \frac{r_{\text{in}} \| Z_1}{(Z_f + r_o) + (r_{\text{in}} \| Z_1)}$$

<u>Proof of assertion (1)</u>: $v_D \approx 0$

Use the approximations: $r_{\text{in}} \gg |Z_1|, |Z_f|$ and $r_{\text{o}} \ll |Z_f|$

$$v_{\text{in}} - v_D \simeq A v_D \frac{Z_1}{Z_f + Z_1} \Rightarrow v_D = \frac{v_{\text{in}}}{1 + BA} \quad \text{so for } BA >> 1 \quad v_D \simeq 0$$

<u>Proof of assertion (2)</u>: $\iota_D \simeq 0$

$$i_{\text{in}} = \frac{v_D}{r_{\text{in}}} = \frac{v_{\text{in}}}{(1+BA) r_{\text{in}}} \to 0 \quad \text{for} \quad BA \gg 1$$

r_{in} is effectively "boot strapped" so that its effective impedance $r_{\text{in}}^{\text{eff}} = r_{\text{in}}(1+BA) \gg r_{\text{in}}$

<u>Proof of assertion (3)</u>: $Z_{\text{out}} \simeq 0$ by Thevenin's theorem

open circuit: $v_{\text{o}}^{\text{o.c.}} = A v_D = -v_{\text{in}} \dfrac{A}{1+BA}$

short circuit: no feedback to input signal, so

$$v_D = v_{\text{in}} \Rightarrow \iota_{\text{o}}^{\text{s.c.}} = \frac{A v_{\text{in}}}{r_o}$$

$$\Rightarrow Z_{\text{out}} = \frac{r_o}{1+BA} \to 0 \quad \text{for} \quad BA \gg 1$$

Closed loop gain in this configuration is:

$$G = \frac{v_o}{v_{\text{in}}} = \frac{A v_d}{v_{\text{in}}} = \frac{A}{1+BA} \to \frac{1}{B} = 1 + \frac{Z_f}{Z_1}$$

We have seen that the use of negative feedback improved the desirable characteristics of an amplifier in several ways:

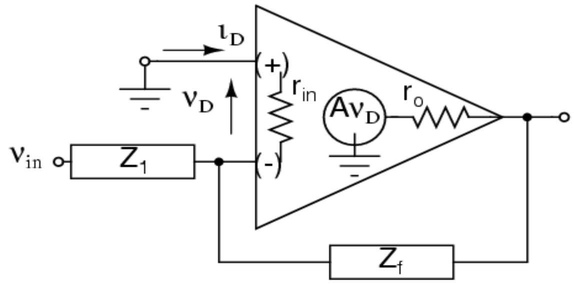
1) increases input impedance
2) reduces output impedance
3) provides a more linear response

The preceding analysis was for small signals, but it still holds if we can neglect the small imperfections of the OPAMP (small DC bias currents into the OPAMP, and small voltage offsets at the inputs).

Next we turn to an analysis of OPAMP circuits using the two most important golden rules (valid when negative feedback is used):

I) The opamp's output attempts to do whatever is needed to make the voltage differences between the inputs equal zero (i.e. $v_{(-)} = v_{(+)}$)
II) The inputs to the opamp draw no current

We will use these simple rules to derive the closed circuit gains for the shunt and series circuits above. We will now drop the small signal notation unless it is explicitly required.

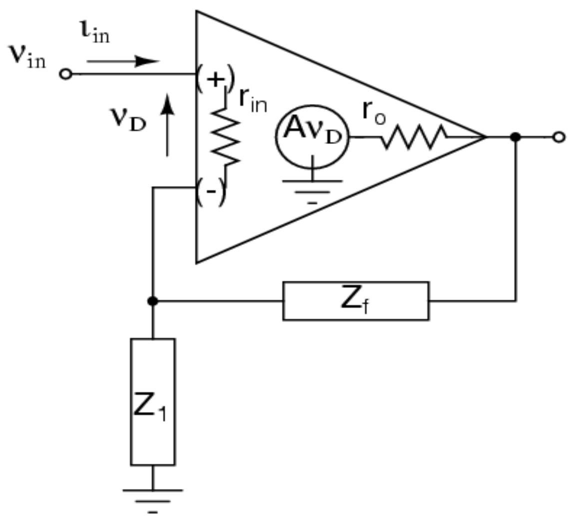


Voltage shunt feedback:

By rule (I): $V_{(-)} = V_{(+)} = 0$

By rule (II): $I_1 = \dfrac{V_{in} - 0}{Z_1} = I_f = \dfrac{0 - V_{out}}{Z_f}$

Therefore $V_{out} = -V_{in} \dfrac{Z_f}{Z_1}$



Voltage series feedback:

By rule (I): $V_{(-)} = V_{(+)} = V_{in}$

By rule (II): $I_1 = \dfrac{V_{in}}{Z_1} = I_f = \dfrac{V_{out} - V_{in}}{Z_f}$

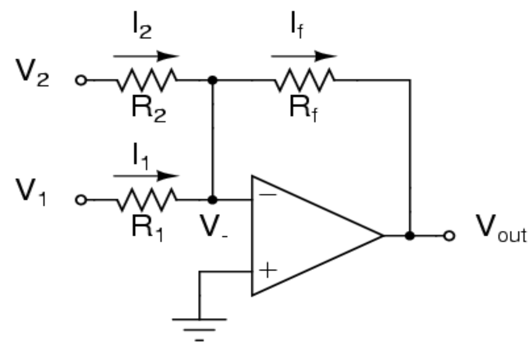
$$V_{in}\left(\frac{1}{Z_1} + \frac{1}{Z_f}\right) = \frac{V_{out}}{Z_f}$$

therefore $V_{out} = V_{in}\left(1 + \dfrac{Z_f}{Z_1}\right)$

Notice: these solutions are general. As long as there is a closed circuit providing negative feedback, the Golden Rules will apply. Subject to this condition arbitrary circuits can be placed into the feedback loop.

Some applications of OPAMPs

Analog adder



Rule 1) $V_- = 0$

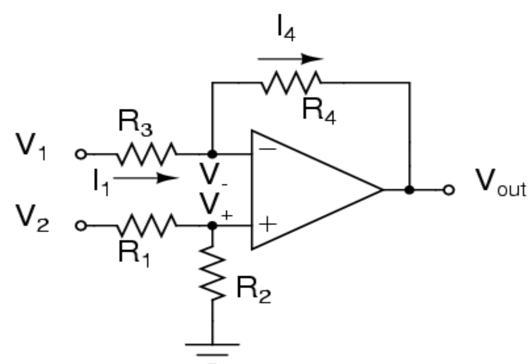
$I_1 = V_1 / R_1$ and $I_2 = V_2 / R_2$

Rule 2) $I_f = I_1 + I_2$

$$V_{\text{out}} = V_- - I_f R_f = -\left[V_1 \frac{R_f}{R_1} + V_2 \frac{R_f}{R_2} \right]$$

Adds, but also inverts

Differential Amplifier



Rule 1)

$$V_- = V_+ = V_2 \frac{R_2}{R_1 + R_2}$$

$$I_1 = \frac{1}{R_3} \left(V_1 - V_2 \frac{R_2}{R_1 + R_2} \right)$$

Rule 2) $I_1 = I_4 \equiv I$

$$V_{\text{out}} = V_- - I R_4 = V_2 \frac{R_2}{R_1 + R_2} \left(1 + \frac{R_4}{R_3} \right) - \frac{R_4}{R_3} V_1$$

$$= \underbrace{\frac{1}{2} \left[\frac{R_2}{R_1 + R_2} (1 + \frac{R_4}{R_3}) + \frac{R_4}{R_3} \right]}_{G_{\text{DIF}}} V_{\text{DIF}} + \underbrace{\left[\frac{R_2}{R_1 + R_2} (1 + \frac{R_4}{R_3}) - \frac{R_4}{R_3} \right]}_{G_{\text{CM}}} V_{\text{CM}}$$

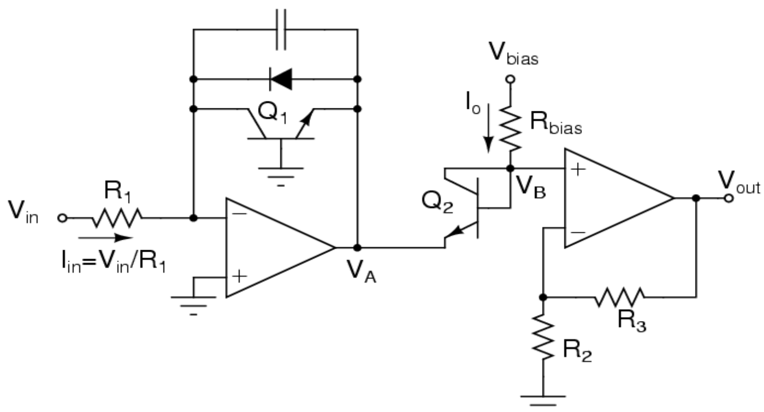
Setting $\left[\frac{R_2}{R_1 + R_2} (1 + \frac{R_4}{R_3}) = \frac{R_4}{R_3} \right]$

i.e. $R_2 / R_1 = R_4 / R_3$

gives $G_{\text{CM}} = 0$ and $G_{\text{DIF}} = \frac{R_2}{R_1}$

this requires precise resistors, difficult to achieve in practice

Logarithmic amplifier



Used to "compress" a signal that varies over many orders of magnitude into a few volt range.

To analyze this circuit:
1) Don't panic

2) Find V_{out}, first find V_A, V_B

$$V_A = -V_{BE}^{(1)} \simeq -V_T \ln \frac{I_{in}}{I_S^{(1)}}$$ from Ebers Moll Eqn.

$$V_B = -V_{BE}^{(1)} + V_{BE}^{(2)} = -V_T \ln\left[\frac{I_{in}}{I_S^{(1)}} \frac{I_S^{(2)}}{I_0}\right] = -V_T \ln\left[\frac{I_{in}}{I_0}\right]$$

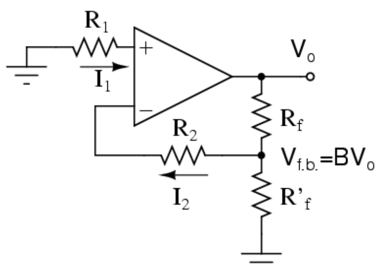
Assuming Q_1 and Q_2 are identical, I_S cancels...

3) The gain for the second OPAMP is $1+\frac{R_3}{R_2}$

then $V_{out} = -V_T \ln\left(\frac{I_{in}}{I_0}\right)\left[1+\frac{R_3}{R_2}\right] \propto \ln V_{in}$

Note: since $V_T = kT/q$ the gain is temperature dependent. A solution is to put a temperature variable resistor in series w/ R_2 $\left(\frac{dR_S}{dT} = \alpha R_S\right)$.

OPAMPS II: Imperfections of OPAMPs



Three D.C. imperfections are common to OPAMPs

- Offset voltage V_{OS}: $V_{out} = A(V_D - V_{OS})$
- Input bias current I_b: $I_b = 1/2(I_1+I_2)$
- Bias current offset I_{OS}: $I_{OS} = (I_1-I_2)$

For the configuration shown:
define $R_B = R_f \| R'_f$

$$V_D = V_{(+)} - V_{(-)} = -I_1 R_1 - (BV_{out} - I_2(R_B+R_2))$$

$$V_o = \frac{1}{B}[V_{OS} - I_1 R_1 + I_2(R_B+R_2)] \text{ for } AB \gg 1$$

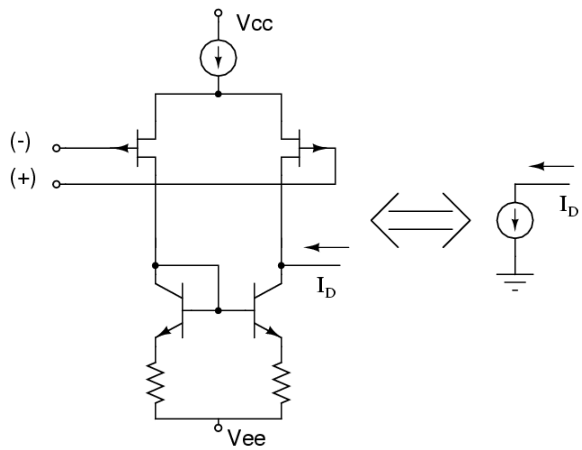
$$= \frac{1}{B}[V_{OS} + I_b(R_B+R_2-R_1) - \frac{1}{2} I_{OS}(R_1+R_2+R_B)]$$

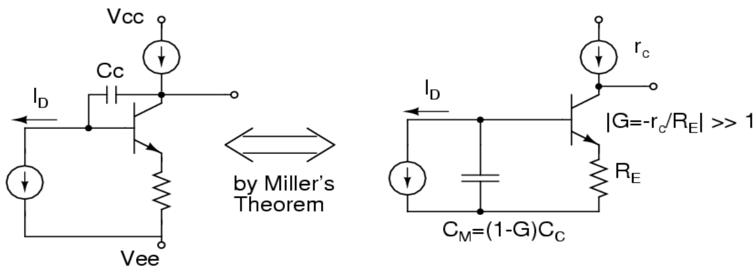
To measure V_{OS}: Choose $R_B + R_2 \simeq R_1$ and $R_1 + R_2 + R_B$ as small as possible Also choose 1/B ~ 1000 so $V_{OS} = BV_{out}$

To measure I_b, I_{OS}: Choose $R_1 \simeq R_2$ so large that $I_n R_n \gg V_{OS}$. (usually 10Meg is good enough for a bipolar OPAMP). Also make $R_B \ll 10\, M\Omega$ for simplicity. Then

i) $I_{OS} = -2BV_{out}/(R_1+R_2)$

ii) Short across R_1, then $I_b = (BV_{out} + \frac{1}{2} I_{OS} R_2)/R_2$

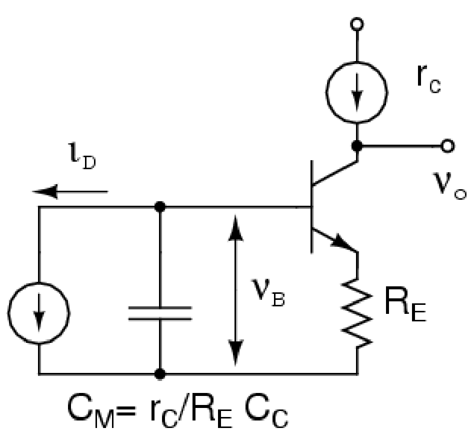




$$\frac{dV_o}{dt} \simeq G\frac{I_D}{C_M} \simeq \frac{I_D}{C_C} = (V_{(+)} - V_{(-)})\frac{g_m}{C_C}$$

Slew rate $\equiv max\left|\frac{dV_o}{dt}\right| = \frac{I_s}{C_C}$ Max rate at which output voltage changes

In order to achieve $\frac{dV_o}{dt} \simeq$ Slew Rate there must be a large difference voltage, V_{DIF}.
This is not the region in which the amplifier operates when negative feedback is used.

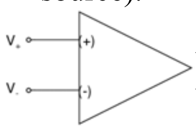


OPAMPS III: Comparators, Oscillators, etc.

The ***comparator*** is a non-linear circuit similar to an OPAMP: Large gain, ~high input Z, differential amp front end.

Differences:

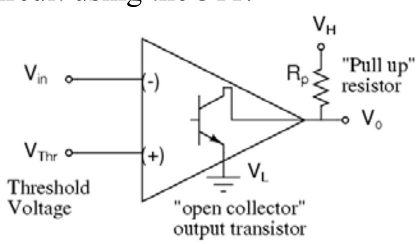
- No "Miller Compensation" capacitor, this means large gain even at high frequency.
- This device is not meant to be used with negative feedback since it would be unstable.
- Output impedance not necessarily small, sometime "open collector" out (a current source).

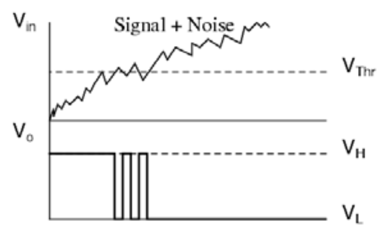


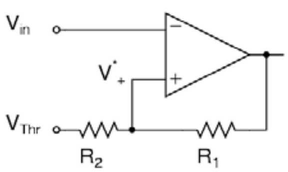
$V_+ > V_-$ $V_o = V_H$ high level (usually V_H, V_L
$V_+ < V_-$ $V_o = V_L$ low level are power supply "rails")

Comparators produce a binary output (V_H or V_L) and are used to interface analog signals (eg from a sensor) to a digital system (like a computer).

A simple circuit using the 311.



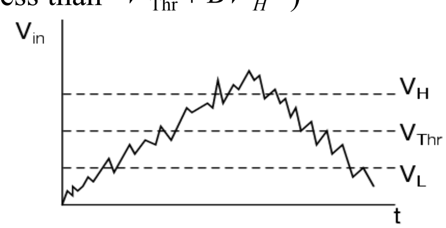


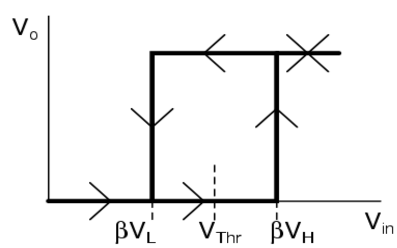


Multiple transitions can be cured by hysteresis --- making the threshold dependent on V_{out} Suppose $V_{in}<V_{Thr}$ and increasing

"True threshold" = $V^*_+ = V_{Thr}(1-B) + VB_o$ where $B = \frac{R_2}{(R_1+R_2)}$

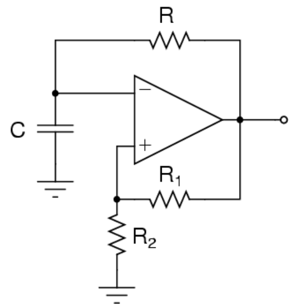
Suppose initially $V_{in}<V_{Thr}$ and is increasing: $V_o = V_H$ and $V^*_+ = V_{Thr} + BV_H$ When V_{in} exceeds V^*_+ , $V_o \rightarrow V_L$ and $V_{in} - V^*_+$ is now a larger difference than previously (ie positive feedback). Output will not switch back to $V_o = V_H$ until $V_{in} < V_{Thr} + BV_L$ (which is less than $V_{Thr} + BV_H$)





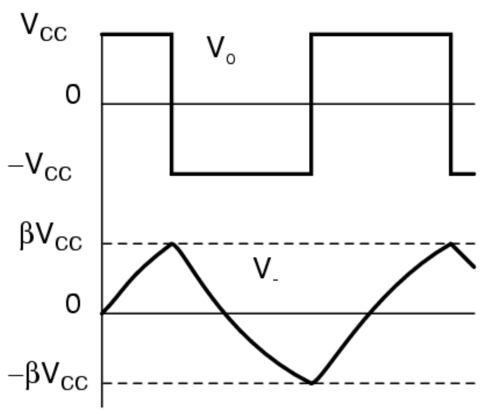
Make $B(V_H-V_L)$ larger than noise to avoid multiple transitions

Relaxation oscillator – square wave output



Assume: $V_H = +V_{CC} = +15V$
$V_L = -V_{CC} = -15V$

Suppose at t=0, $V_- = V_{cap} = 0$, $V_o = +15V$
Capacitor will charge towards $+V_{CC}$ w/ time constant RC. When $V_- > BV_{CC}$, V_o goes to $-V_{CC}$ and the capacitor charges towards $-V_{CC}$ w/ time constant RC. When $V_- < BV_{CC}$, V_o goes to $+V_{CC}$ and the cycle repeats. $B = \frac{R_2}{(R_1 + R_2)}$



The half period $T_{1/2}$ is given by :

$$V_{CC}(1+B)(1-e^{T_{1/2}/RC}) = 2BV_{CC}$$

$$\frac{(1-B)}{(1+B)} = e^{T_{1/2}/RC}$$

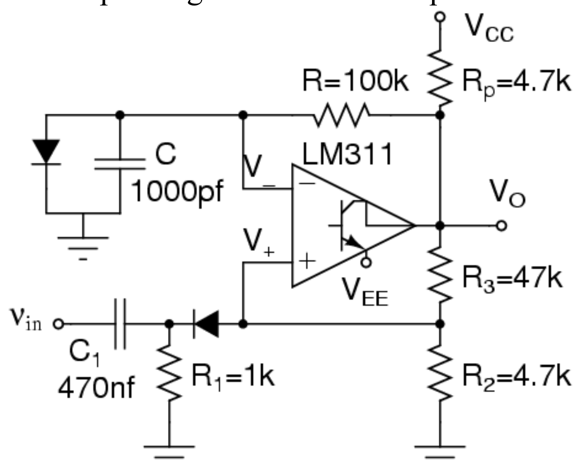
Full period = 2 $T_{1/2}$, $T = 2RC \ \ln\left(\frac{1+B}{1-B}\right)$

For a 311 the square wave is slightly asymmetric. Z_{out}=pullup R when $V_O = V_H$ and $Z_{out} \sim 0$ when $V_o = V_L$. This it spends more time at $V_o = V_H$

One-shot or Monostable

The relaxation oscillator was astable (no stable state). The circuit below has one stable state to which it returns after spending a brief time in a quasi-stable state.

$V_{CC}=-V_{EE}=15V$



$$V\_=V_D\simeq 0.6\,V\,(D_1, D_2\,\text{on})$$

Initially take $V_O=V_{CC}$ then $V_{\text{Thr}}=V_+-V_D=\dfrac{V_{CC}-V_D(1+R_3/R_2)}{1+R_3/R_1+R_3/R_2}$

When $-\nu_{\text{in}}>V_{\text{Thr}}$ then $V_O\rightarrow V_{EE}$ D_1,D_2 shutoff, V_ changes towards V_{EE} and $V_+=\dfrac{R_2}{R_2+R_3}V_{EE}=BV_{EE}$. When V_<V_+ then $V_O\rightarrow V_{CC}$ and the device returns to its initial state.