## Important mathematics addendum for PHYS 321 HW #9, Prob. 3

You will need to do certain integrals involving Legendre polynomials. These are of the form

$$\begin{aligned}\int_0^\pi d\theta \sin\theta \cos^2\theta \, P_2(\cos\theta) &\equiv \int_{-1}^1 dt\, t^2 P_2(t) \\ &= \int_{-1}^1 dt \left[\tfrac{2}{3}P_2(t) + \tfrac{1}{3}\right] P_2(t) \\ &= \tfrac{2}{3}\int_{-1}^1 dt \left[P_2(t)\right]^2 = \frac{4}{15}\end{aligned} \tag{1}$$

$$\int d\hat{r}\, P_\lambda(\hat{r}\cdot\hat{z}) P_{\lambda'}(\hat{r}\cdot\hat{R}) = \delta_{\lambda\lambda'}\left(\frac{4\pi}{2\lambda+1}\right) P_\lambda(\hat{z}\cdot\hat{R}) \ . \tag{2}$$

Don't forget that $\sin^2\theta = 1 - \cos^2\theta$ !!

The first three Legendre polynomials are

$$P_0(t) = 1$$
$$P_1(t) = t$$
$$P_2(t) = \tfrac{3}{2}t^2 - \tfrac{1}{2}$$

**Hint:** It is best to compute the potential energy as a function of the angle between the plane of the orbit and the Earth's axis of rotation. The torque is then given by

$$N_\theta = -\frac{\partial V}{\partial\theta} \, .$$