

## PHYS 321 Homework Assignment

### Due: Friday, 6 September 2002

1. Having deduced that gravitational acceleration falls off as the inverse square of distance, Newton wrote the following formula for the gravitational acceleration of an object near the Earth's surface (*e.g.* a falling apple):

$$g = \frac{K}{R_E^2}.$$

He then noted that the Moon, in a nearly circular orbit of radius  $R_{E-M}$ , should accelerate toward the Earth with acceleration

$$a_M = \frac{K}{R_{E-M}^2},$$

where  $R_{E-M} \approx 60R_E$ . Finally, he noted that the centripetal acceleration of an object moving at constant speed  $v$  in a circular orbit is  $v^2/R$ ; this permitted him to compare the Moon's actual acceleration with that predicted from the inverse-square law of gravitation. (The Moon's orbital period is about 27.5 days, and the Earth's radius is about 4000 miles. In these English units,  $g = 32 \text{ ft} - \text{sec}^{-2}$ .)

Please emulate Sir Isaac and compute the ratio of  $a_M$  to  $g$  both from Newton's theory, and using the formula for centripetal force (kinematics!!), also by Newton, putting in the known values of distance  $R_{E-M}$  and orbital period  $T_M$ .

#### **Solution:**

Since  $R_{E-M} \approx 60R_E$ , Newton's Law of Universal Gravitation predicts

$$a_M \approx \frac{a_M}{g} \approx \frac{1}{3600}.$$

On the other hand, the centripetal acceleration is

$$\frac{v^2}{R_{E-M}} = \frac{(2\pi R_{E-M}/T_M)^2}{R_{E-M}} = (2\pi)^2 \frac{R_{E-M}}{T_M^2}$$

and taking  $R_{E-M} = 240,000 \times 5280$  feet and  $T_M = 27.5 \times 24 \times 3600$  sec, we find

$$\frac{v^2}{R_{E-M}} = 0.00886 \text{ ft/sec}$$

whereas  $g/3600 = 0.00894$  ft/sec. Their ratio is 0.991—the agreement is better than 1%.

2. In their discussion of the drag racer in Ch. 1, Barger and Olsson neglect torque. Assume the drag racer of Fig. 1.1 is 15 feet long, and that its rear (driving) wheels are 4 feet in diameter. Where must the center of mass be located (that is, what is  $b_2$  if  $b_1 + b_2 = 15$  ft) for the car to be able to achieve the acceleration of  $1g$ ?

**Solution:**

The balance of forces gives

$$N_1 + N_2 = mg$$

as in Eq. 1.20. The rear tires exert force  $F \leq \mu N_2$  on the pavement. The forward acceleration is, by Newton's 2<sup>nd</sup> Law,

$$a = \frac{F}{m}.$$

Finally, let us balance the torques. Taking the center of the rear tire as the point of rotation we then have

$$mgb_2 - N_1(b_1 + b_2) - RF = 0.$$

These equations (and inequalities) can be rearranged to get

$$g \left( 1 - \frac{b_2}{b_1 + b_2} \right) \leq a \left( \frac{1}{\mu} - \frac{R}{b_1 + b_2} \right).$$

If  $a = g$  and  $\mu = 1$ , then the condition for rotational stability is  $b_2 > 2$  ft.

3. The shepherd's sling (à la David) is about 2.5 ft long, as is the thrower's arm. Assuming a slinger can revolve his arm about his shoulder joint 4 times per second, what is the maximum range he can achieve in the absence of air resistance?

**Solution:**

The speed of an object traveling in a circle of radius 5 feet at angular velocity  $8\pi \text{ sec}^{-1}$  is about 126 ft/sec. Taking  $g = 32.2 \text{ ft-sec}^{-2}$  and

using the fact that the range of a projectile fired at speed  $v$  at angle  $\theta$  from the horizontal is

$$R = \frac{v^2}{g} \sin(2\theta),$$

we find a range of about 490 ft, or roughly 160 yards.

To get the range of a projectile, take  $z$  to be the vertical direction,  $x$  the horizontal. Then

$$\ddot{z} = -g, \quad \ddot{x} = 0$$

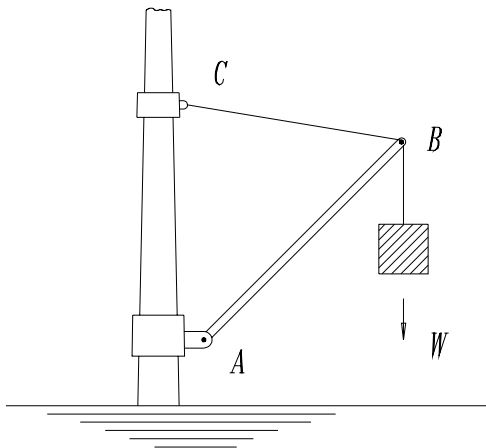
so that with  $z_0 = x_0 = 0$  and  $\dot{z}_0 = v \sin \theta$ ,  $\dot{x}_0 = v \cos \theta$ ,

$$z = x \tan \theta - \frac{g}{2v^2 \cos^2 \theta} x^2$$

or, looking for the two roots of  $z = 0$ , we get  $x = 0$  and

$$x_{\max} = \frac{2v^2 \cos^2 \theta \tan \theta}{g} = \frac{v^2}{g} \sin(2\theta).$$

4. The lifting hoist shown below has a boom held by a cable. For a given weight  $W$ , what is the tension in the cable, and what is the compressive stress in the boom? Express the answers in terms of the distances  $\overline{AB}$ ,  $\overline{AC}$  and  $\overline{BC}$ . Ignore the weights of cable and boom.



**Solution:**

Let  $\varphi$  be the angle between the vertical and the line  $\overline{CB}$ , and  $\theta$  that between the boom  $\overline{AB}$  and the vertical. Then we have the following geometric relations:

$$\overline{AB} \sin \theta = \overline{CB} \sin \varphi$$

and

$$\overline{AB} \cos \theta + \overline{CB} \cos \varphi = \overline{AC}.$$

Resolving the compression force  $F$  in the boom and the tension  $T$  in the cable horizontally and vertically, we have

$$\begin{aligned} -T \sin \varphi + F \sin \theta &= 0 \\ F \cos \theta + T \cos \varphi &= W \end{aligned}$$

These equations can be solved to give

$$F = W \frac{(\overline{AB})}{(\overline{AC})},$$

and

$$T = W \frac{(\overline{CB})}{(\overline{AC})}.$$

5. (Barger & Olsson, prob. 1.11) A ball of mass  $m$  is thrown vertically upward with speed  $v_i$ . If the air resistance is proportional to  $v^2$  and the terminal speed (for an object dropped from an infinite height) is  $v_t$ , show that the ball returns to its initial position with speed  $v_f$  satisfying

$$\frac{1}{v_f^2} = \frac{1}{v_i^2} + \frac{1}{v_t^2}.$$

**Solution:**

We follow the discussion in B&O, pp. 9ff. On the way up the equation of motion is

$$\ddot{z} = -g - \gamma \dot{z}^2,$$

and on the way down the last term reverses in sign:

$$\ddot{z} = -g + \gamma \dot{z}^2.$$

Since we are interested in altitude, not time<sup>1</sup>, we change the independent variable from  $t$  to  $z$ , *via* the chain rule: if  $v = \dot{z}$ , then

$$\frac{dv}{dt} \equiv \frac{dv}{dz} \frac{dz}{dt} \equiv v \frac{dv}{dz}$$

so we obtain the equation(s) of motion

$$v \frac{dv}{dz} = -g \mp \gamma v^2.$$

It is worth noting that if the ball could fall arbitrarily far, it would cease to accelerate when it reached terminal speed given by

$$v_t^2 = \frac{g}{\gamma}.$$

The equations of motion are separable, that is, can be rewritten

$$\frac{v dv}{-g \mp \gamma v^2} = dz.$$

Both sides are easily integrated, if we note that  $2v dv = d(v^2)$ , giving

$$-\int_{v_0^2}^0 \frac{du}{u + v_t^2} = 2\gamma \int_0^h dz$$

and

$$-\int_{v_f^2}^0 \frac{du}{v_t^2 - u} = 2\gamma \int_h^0 dz$$

or

$$\int_0^{v_0^2} \frac{du}{u + v_t^2} = 2\gamma \int_0^h dz = \int_0^{v_f^2} \frac{du}{v_t^2 - u}.$$

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<sup>1</sup>... because the ball reaches height  $h$ , where its upward velocity becomes zero, then falls from that height, reaching the initial point with velocity  $\dot{z} = -v_f$

We perform the integrations, obtaining

$$\ln \left( \frac{v_0^2 + v_t^2}{v_t^2} \right) = \ln \left( \frac{v_t^2}{v_t^2 - v_f^2} \right)$$

and after some algebra,

$$\frac{1}{v_f^2} = \frac{1}{v_0^2} + \frac{1}{v_t^2}.$$