PHYS 321 Homework Assignment #10
Due: Friday, 06 December 2002 (5 probs)

1. The lowest resonant frequency of an organ pipe of length $\ell$, closed at both ends, is $\nu = v_s/2\ell$. Use this fact to explain why you would sound like Donald Duck if you were breathing a mixture of 5% O$_2$ and 95% He gas, at a pressure of 4 atmospheres (deep-sea divers’ mixture). Keep in mind that He is a monoatomic gas.

2. It is easy to see that the phase velocity of waves along our model of a line of masses connected by springs is

$$v_{\text{phase}} = \frac{\Omega a}{2 \arcsin(\omega/2\Omega)}$$

where $a$ is the interatomic spacing, $\Omega$ is the harmonic oscillator frequency of one mass on one spring, and $\omega$ is the oscillation frequency of the wave. The group velocity (that is, the speed of a wave packet) is given by

$$v_{\text{group}} = a \frac{d\omega}{d\varphi},$$

where

$$\sin\left(\frac{\varphi}{2}\right) = \frac{\omega}{2\Omega}.$$  

Thus show that as $\omega \to 2\Omega$, the ratio $v_{\text{group}}/v_{\text{phase}} \to 0$.

3. Problem 3-14 on p. 110 of B&O.

4. The block and pendulum shown below are similar to the ones on the midterm, except this time the block is connected to a solid wall by a spring of constant $K$. Write the Lagrangian for this system in appropriate generalized coordinates, then find the generalized momenta, derive the Hamiltonian and write down Hamilton’s equations of motion.
5. For small displacements from equilibrium, show that the mass- and spring-constant matrices for the above system are positive-definite. Find the normal modes and characteristic frequencies of oscillation.