PHYSICS 355 - Practice Test - 50 minuites

1. The harmonic oscillator ladder operators are:

$$a_\pm = \frac{1}{\sqrt{2m}}\,(p \pm im\omega x)$$

   (a) Calculate the commutator, $[a_+, a_-]$.

   (b) Use the ladder operators to derive the unnormalized ground state and first excited state wave functions for the harmonic oscillator.

2. A particle in an infinite square well extending from $x = 0$ to $x = a$ is in the initial state

$$\psi(x) = Ax(a-x), \quad 0 \le x \le a, \qquad \psi(x) = 0, \qquad \text{elsewhere}$$

   (a) Determine A

   (b) From the symmetry of the function write down $\langle x \rangle$ and $\langle p \rangle$ at time $t = 0$. (No calculation is needed)

   (c) Calculate σ_x and σ_p at $t = 0$ and verify that the uncertainty principle is satisfied.

3. A particle in an infinite square well extending from $x = 0$ to $x = a$ is in the initial state

$$\psi(x) = Ax(a-x), \quad 0 \le x \le a, \qquad \psi(x) = 0, \qquad \text{elsewhere}$$

. At $t = 0$, the potential was turned off ($V = 0$ everywhere) and the particle was set free. Find the function $\phi(k)$ needed to represent the solution for later times as

$$\Psi(x,t) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \phi(k) e^{i(kx - \omega t)}\, dk$$

Mathematical formulas

$$e^{ix} = \cos x + i \sin x$$

$$\cos x = \frac{e^{ix} + e^{-ix}}{2}, \qquad \sin x = \frac{e^{ix} - e^{-ix}}{2i}$$

$$\cos^2 x = \frac{1 + \cos 2x}{2}, \qquad \sin^2 x = \frac{1 - \cos 2x}{2}$$

$$\int_{-\infty}^{\infty} e^{-x^2/a^2} \; dx = a\sqrt{\pi}$$

$$\int_0^{\infty} x^n e^{-x} \; dx = n!$$

$$\int x^n \; \sin(ax) \; dx = -\frac{x^n}{a} \cos(ax) + \frac{n}{a} \int x^{n-1} \cos(ax) \, dx$$

$$\int x^n \cos(ax) \; dx = \frac{x^n}{a} \sin(ax) - \frac{n}{a} \int x^{n-1} \sin(ax) \, dx$$

$$\int x^n \, e^{\alpha x} \, dx = \frac{x^n}{\alpha} e^{\alpha x} - \frac{n}{\alpha} \int x^{n-1} \, e^{\alpha x} \, dx$$