Phys 356 Final Exam May 6, 2006

**Instructions:**

This is an in class, three hour exam. You may refer to your textbook, class notes, and homework solutions, and you may use a calculator if desired. No other reference materials are permitted. Turn in your solutions stapled to these exam sheets.

The exam consists of six problems, each worth 10 points. For full credit, you must explain your reasoning and show all your work. You may cite the result of a calculation in the text, notes, or homework without rederiving it, but state clearly where the result is from.

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1. (a) Consider $N$ identical bosonic particles, each of which has two internal states $\psi_a$ and $\psi_b$. The particles start in a many-body state with one of the particles in state $\psi_a$ and all the rest in state $\psi_b$. Write this state out correctly, and call it $\Psi_A$. You can use the notation $\psi_j(n)$ to indicate the single-particle state $j$ for particle $n$.

(b) A perturbation $H'(t)$ drives the transition from $\Psi_A$ to the state

$$\Psi_B \equiv \psi_b(1)\psi_b(2)\ldots\psi_b(N)$$

The perturbation has the form $H'_{(1)} + H'_{(2)} + \ldots + H'_{(N)}$ where $H'_{(n)}$ acts only on particle $n$. For each single particle, the perturbation has a matrix element $H'_{ab}(t) = \langle\psi_a(n)|H'_{(n)}|\psi_b(n)\rangle$ that is (by symmetry) independent of $n$. If there were *only* one particle, this would give a transition probability

$$P_{a\to b} = \frac{1}{\hbar^2}\left|\int_0^t H'_{ba}(t')e^{i\omega_0 t'}\,dt'\right|^2$$

Your job is to calculate the transition probability $P_{A\to B}$ for the $N$-particle system and compare it to the single particle result $P_{a\to b}$.

2. Use the variational method to estimate the ground state energy of the linear potential $V(x) = \alpha|x|$, using a trial wave function $\psi(x) = A\exp(-|x|/a)$.

3. Using the Born approximation, calculate the scattering amplitude for the potential

$$V(\mathbf{r}) = \begin{cases} V_0 & \text{if } -a < x < a,\ -a < y < a, \text{ and } -a < z < a \\ 0 & \text{otherwise} \end{cases}$$

for arbitrary incident energy $E$. This problem is easiest to work in Cartesian coordinates, but you should indicate clearly how your answer depends on the scattering angles $\theta$ and $\phi$.

4. Suppose a hydrogen atom in its ground state is placed in a static electric potential $V = \mathcal{E}z$. Use perturbation theory to calculate the energy shift to second order, including only states up to $n = 3$ in the second order correction. You can neglect the fine and hyperfine structure of the atom.

Hint: there are only two nonzero integrals in this problem, and I did one of them for you in class.

5. A spin-1/2 electron is in the $m = +1/2$ state in a static magnetic field $\mathbf{B} = B_0\hat{\mathbf{z}}$. At time zero, a transverse field $B_1\hat{\mathbf{x}}$ is turned on, and then turned off again at time $t$. What is the probablity that the electron is now in the $m = -1/2$ state?

6. A particle of mass $m$ is in the ground state of a one-dimensional infinite square well of dimension $a$. After the size of the well is adiabatically increased to $2a$, what is the expectation value of particle's energy?