Adiabatic Approx

- Imagine pendulum ⟶ if you moved it gently ⟶ steady, you would not see a change.

- gradual change is called Adiabatic.

Two Parameters: T_i :- internal time i.e. like period of osc.

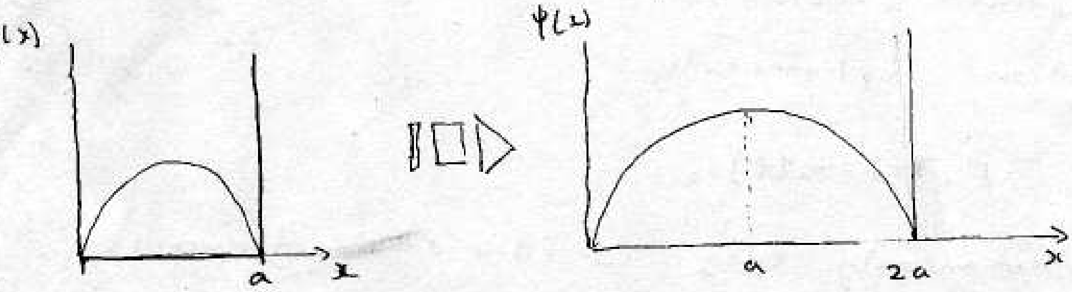
T_e :- external time i.e. time over which the parameters change noticeably.

eg if $T_p = 2\pi\sqrt{L/g}$

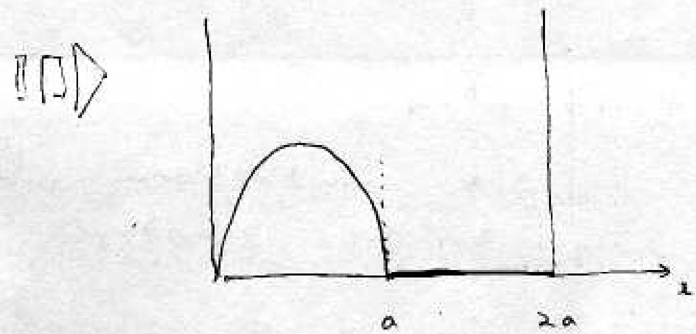
$\Rightarrow T_p = 2\pi\sqrt{\dfrac{L(t)}{g}}$

Adiabatic theorem says: Assuming non degeneracy

nth eigen state of H^i ⟶ nth eigen state of H^f



Slow



Fast

* Energy Is not conserved. t is not Integral of the Motion

$$\psi^i(x) = \sqrt{\frac{2}{a}} \sin\left[\frac{\pi}{a} x\right]$$

$$\psi^f(x) = \sqrt{\frac{2}{2(a)}} \sin\left[\frac{\pi x}{2a}\right]$$

If solve first then $\psi^f(x) = \sum_n c_n(t,x)\, \psi(x,t)$

ψ^f would be some complicated linear combo of the H^f |

$\Rightarrow$ energy is conserved.

Proof starting out with the particle in some nth eigen state.

$$\hat{H}\psi_n = E_n \psi_n$$

by separation of variables we get:

$$\psi_n(t) = \psi_n e^{\frac{-iE_n t}{\hbar}}$$

If the hamiltonian changes with time then so does the eigen function instantaneously.

1] $\hat{H}(t)\psi_n(t) = E_n(t)\,\psi_n(t)$

but also instantaneously they constitute an orthonormal set of vectors

$$\langle \psi_n(t) | \psi_m(t) \rangle = \delta_{nm}$$

They are also complete and form part thus of the general solution to the Schrodinger eq.

2] $i\hbar \frac{\partial}{\partial t}\psi(t) = \hat{H}(t)\,\psi(t)$

from ch 2. p 25. from separation of variables yielded:

$$\Psi(x,t) = \psi(x)\,\varphi(t) \Rightarrow \Psi(x,t) = \sum_n c_n \psi_n(x)\,\varphi_n(t).$$

$$\Rightarrow \quad i\frac{d\varphi}{dt} = -\frac{iE\varphi}{\hbar}$$

$$\Rightarrow \quad \frac{d\varphi}{\varphi} = -\frac{i}{\hbar}E\,dt$$

$$\int \frac{d\varphi}{\varphi} = \int \frac{-i}{\hbar}E\,dt$$

normally $E \neq E(t)$

But now $E = E(t)$

$$\ln\varphi = -\frac{i}{\hbar}\int E(t)\,dt + K$$

$$\varphi = e^{\phi}\underbrace{e^{-\frac{i}{\hbar}\int E(t)\,dt}}$$

general time dep phase $\varphi_n(t)$

$$\Psi(x,t) = \sum_i c_n(t)\,\psi_n(t)\,e^{-\frac{i}{\hbar}\int_0^t E(t)\,dt}$$

3] $$= \sum_i c_n(t)\,\psi_n(t)\,e^{i\theta_n} \quad ; \quad \theta_n = \frac{-1}{\hbar}\int_0^t E_n(t')\,dt'$$

Could lump the $e^{i\theta_n}$ to the $c_n(t)$ but leave it like this since it would also be present in the time Indep case.

$$\Psi(x,t) = \sum_n c_n(t)\,\psi_n(t)\,e^{i\theta_n(t)} \quad \text{into} \quad 2]$$

$$\Rightarrow \quad i\hbar \frac{\partial}{\partial t}\Psi(t) = \hat{H}(t)\,\Psi(t)$$

$$i\hbar \frac{\partial}{\partial t} \sum_n c_n(t)\psi_n(t) e^{i\theta_n(t)} = \hat{H}(t) \sum_n c_n(t)\psi_n(t) e^{i\theta_n(t)}$$

$$i\hbar \sum_n \left[\dot{c}_n \psi_n e^{i\theta_n} + \frac{\partial}{\partial t}\left[\psi_n e^{i\theta_n(t)}\right] c_n \right] = \sum_n c_n (\hat{H}\psi_n) e^{i\theta_n}$$

$$i\hbar \sum_n \left[\dot{c}_n \psi_n e^{i\theta_n} + c_n\left[\dot{\psi}_n e^{i\theta_n} + i e^{i\theta_n}\dot{\theta}_n \psi_n\right] \right] = \sum_n c_n(\hat{H}\psi_n) e^{i\theta_n}$$

$$i\hbar \sum_n \left[\dot{c}_n \psi_n + c_n \dot{\psi}_n + i c_n \psi_n \dot{\theta}_n \right] e^{i\theta_n} = \sum_n c_n(\hat{H}\psi_n) e^{i\theta_n}$$

use $\hat{H}\psi_n = E_n \psi_n$

$$i\hbar \sum_n \left[\dot{c}_n \psi_n + c_n \dot{\psi}_n + i c_n \psi_n \frac{\partial}{\partial t}\left[\frac{-1}{\hbar}\int_0^t E_n(t')\,dt'\right]\right] e^{i\theta_n} = \sum_n c_n(\hat{H}\psi_n) e^{i\theta_n}$$

$$i\hbar \sum_n \left[\dot{c}_n \psi_n + c_n \dot{\psi} + i c_n \psi_n \frac{-1}{\hbar} E_n \right] e^{i\theta_n} = \sum_n c_n E_n \psi_n e^{i\theta_n}$$

$$\sum_n \left[i\hbar \dot{c}_n \psi_n e^{i\theta_n} + i\hbar c_n \dot{\psi}_n e^{i\theta_n} + \cancel{c_n \psi_n E_n e^{i\theta_n}} \right] = \cancel{\sum_n c_n E_n \psi_n e^{i\theta_n}}$$

$$\sum_n \cancel{i\hbar}\, \dot{c}_n \psi_n e^{i\theta_n} = -\cancel{i\hbar} \sum_n c_n \dot{\psi}_n e^{i\theta_n}$$

$$\sum_n \dot{c}_n \psi_n e^{i\theta_n} = -\sum_n c_n \dot{\psi}_n e^{i\theta_n}$$

$$\psi_m \sum_n{}' \dot{c}_n \psi_n e^{i\theta_n} = -\psi_m \sum_n{}' c_n \dot{\psi}_n e^{i\theta_n}$$

$$\sum_n{}' \dot{c}_n \psi_m \psi_n e^{i\theta_n} = -\sum_n{}' c_n \psi_m \dot{\psi}_n e^{i\theta_n}$$

$$\sum_n{}' \dot{c}_n \langle \psi_m | \psi_n \rangle e^{i\theta_n} = -\sum_n{}' c_n \langle \psi_m | \dot{\psi}_n \rangle e^{i\theta_n}$$

collapses sum $$\sum_n{}' \dot{c}_n \delta_{mn} e^{i\theta_n} = -\sum{}' c_n \langle \psi_m | \dot{\psi}_n \rangle e^{i\theta_n}$$

$$\dot{c}_m e^{i\theta_m} = -\sum{}' c_n \langle \psi_m | \dot{\psi}_n \rangle e^{i\theta_n}$$

4] $$\dot{c}_m(t) = -\sum{}' c_n \langle \psi_m | \dot{\psi}_n \rangle e^{i(\theta_n - \theta_m)}$$

$H(t)\,\psi_n(t) = E_n(t)\,\psi_n(t)$

$\hat{H}|n\rangle = E_n |n\rangle$

Differentiating it using 1] $$\frac{d}{dt}\hat{H}|n\rangle = \frac{d}{dt}E_n|n\rangle$$

$$\dot{\hat{H}}|n\rangle + \hat{H}|\dot{n}\rangle = \dot{E}_n|n\rangle + E_n|\dot{n}\rangle$$

$(ABC)^+$
$(C^+B^+A^+)$

$$\langle m|\dot{\hat{H}}|n\rangle + \langle m|\hat{H}|\dot{n}\rangle = \langle m|\dot{E}_n|n\rangle + \langle m|E_n|\dot{n}\rangle$$

$$\langle m|\dot{\hat{H}}|n\rangle + \langle m|\hat{H}|\dot{n}\rangle = \langle m|\dot{E}_n|n\rangle + E_n\langle m|\dot{n}\rangle$$

// $\langle m|H|\dot{n}\rangle$ using hermiticity

$= (\langle \dot{n}|H^+|m\rangle)^+$ $$\Rightarrow \langle m|\dot{\hat{H}}|n\rangle + E_m\langle m|\dot{n}\rangle = \dot{E}_n\langle m|n\rangle + E_n\langle m|\dot{n}\rangle$$

$= (\langle \dot{n}|H|m\rangle)^+$

$= E_m^+(\langle \dot{n}|m\rangle)^+$ $$\langle m|\dot{\hat{H}}|n\rangle = \dot{E}_n\delta_{mn} + (E_n - E_m)\langle m|\dot{n}\rangle$$

$= E_m\langle m|\dot{n}\rangle$

$$\langle \psi_m | \dot H | \psi_n \rangle = (E_n - E_m) \langle \psi_m | \dot\psi_n \rangle$$

$$\langle m | \dot H | n \rangle = (E_n - E_m) \langle m | \dot n \rangle \qquad m \neq n$$

use 4] writting out the sum the $n = m$

$$\dot c_m(t) = -c_m \langle m | \dot m \rangle - \sum_{n \neq m} c_n \frac{\langle m | \dot H | n \rangle}{(E_n - E_m)} e^{i[\cdots]}$$

$$[\ldots] = \theta_n - \theta_m$$

$$= -\frac{1}{\hbar} \left[\int_0^t E_n(t')\,dt' - \int_0^t E_m(t')\,dt' \right]$$

$$= -\frac{1}{\hbar} \left[\int_0^t E_n(t') - E_m(t')\,dt \right]$$

$$\dot c_m(t) = -c_m \langle m | \dot m \rangle - \underbrace{\sum_{n \neq m} c_n \frac{\langle m | \dot H | n \rangle}{(E_n - E_m)} e^{-\frac{i}{\hbar}\int_0^t [E_n(t') - E_m(t')]dt}}_{\text{small ignore}}$$

<u>Exact</u> <u>Result</u>.

so assume $\dot H$ is small ie $\ll E_n - E_m$

ie over one some Time $\ll T_i$

$$\dot c_m(t) = -c_m \langle m | \dot m \rangle$$

$$\dot c_m(t) = -c_m(t) \langle \psi_m(t) | \dot\psi_m(t) \rangle$$

$\dot{c} = -c \langle m | \dot{m} \rangle$ took out sub m's

$$\frac{dc}{dt} = -c \langle m | \dot{m} \rangle$$

$$\frac{dc}{c} = - \langle m | \dot{m} \rangle$$

$$\int_{c(0)}^{c(t)} \frac{dc}{c} = - \int_0^t \langle m | \dot{m} \rangle \, dt$$

$$\ln c(t) - \ln c(0) = - \int_0^t \langle m | \dot{m} \rangle \, dt$$

$$c(t) = c(0) e^{-\int_0^t \langle m | \dot{m} \rangle dt}$$

$$\Rightarrow \boxed{\begin{aligned} \gamma_m(t) &\equiv i \int_0^t \langle m | \dot{m} \rangle \, dt \\ c_m(t) &= c_m(0) e^{i \gamma_m(t)} \end{aligned}}$$

so if particle starts out in the mth eigen state then it remains in the mth eigen state

- only picks up phase factors
- once you make the approx → no more coupling to excited states

$$\Psi(t) = \sum_n \ldots \quad \text{if } c_n(0) = 1, \; c_m(0) = 0 \quad m \neq n$$

ure 3]

$$\boxed{\Psi_n(t) = e^{i \gamma_n(t)} e^{i \theta_n(t)} \psi_n(t)}$$