

Lecture 39

Recap -

Spontaneous emission: excited states have finite lifetimes, eventually make spontaneous transition to ground state

Adiabatic approximation: if H changes slowly enough, system stays in same state while state itself changes.

Rest of the course -

no more hard calculations, say something about what QM means.

Basic question: what exactly is a wavefunction?

Can't observe it directly, behaves in some strange ways.

Today focus on one strange thing: measurement

Suppose we measure something about a particle in state ψ
To be concrete, let's measure energy.

QM says: to predict results, first write ψ as sum of eigenstates of energy operator (H)

$$\psi = \sum_n c_n \psi_n \quad \text{with } H\psi_n = E_n \psi_n$$

Then measurement gives result E_n with prob $|c_n|^2$

Also, after measurement, system is in state ψ_n

Say that wave function "collapses" to ψ_n

Clear enough, and agrees well with experiments.

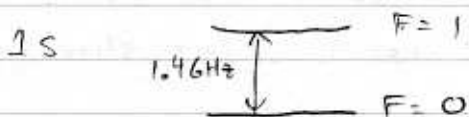
But it is kind of strange.

Evolution of ψ normally governed by Schr. Egn,
but Schr. Egn. doesn't have "collapse" in it.

- Seems inelegant to need two separate types of evolution
- Also, what constitutes a measurement? When exactly do we need to impose this "collapse"?

To get a better idea, consider a measurement example.

Recall hydrogen atom has two ground states due to hyperfine effect



Upper state does decay
via spontaneous emission,
but lifetime is ~ 40000 years.
- stable for our purposes

Can prepare an atom in a superposition of ψ_0 & ψ_1
by applying B-field oscillating at 1.4 GHz

(Need B, not E, because we're affecting spin
instead of charge.)

If we start out in ψ_0 , and apply resonant field $\Delta=0$
have

$$c_1(t) = -i \sin \frac{\omega t}{2} \quad \omega \propto B$$

In particular, apply for time $t_1 = \frac{2}{\omega} \cdot \frac{\pi}{4}$

After pulse, $\psi(t) = \frac{1}{\sqrt{2}} (\psi_0 - i \psi_1)$

Now, can measure state by applying laser tuned between $F=1$ state and $2p$ excited state.

Set laser strength & duration to drive transition with 100% probability

Then wait several ns. If atom in $2p$ state it will decay, emitting a photon.

So if we detect photon, know atom was in $F=1$ state. Furthermore, if we make laser with correct polarization, we can be sure $2p$ state decayed back to $F=1$ state.

So if we detect photon, know atom is in 2_1 , if no photon, know atom in 2_0 .

That is our measurement.

Question is, what makes it a measurement?

Some possible answers:

A: Fact that I , a conscious being, learned the state of the system. Important part is the observer.

B: Fact that the state of my detector changed depending on the state of the atom. Important part is interaction with a macroscopic object.

C: Fact that atom scattered a photon at all. Important part is interaction with anything.

All of these have some merit, but also some problems:

A: Does that mean the universe would behave totally differently if humans weren't around?

B: How big does something have to be to count as macroscopic?

C: But photons are quantum entities. Shouldn't the Schr. eqn handle this by itself?

We can imagine an experiment to resolve these options.

Sequence:

I) Prepare atom in ψ_0

P1 Apply B-field pulse $\psi \rightarrow \frac{1}{\sqrt{2}}(\psi_0 - i\psi_1)$

M1 Attempt measurement in various ways

P2 Apply B-field again

M2 Measure again in straight forward way.

Two possibilities for M2:

- If M1 is not a successful measurement, then ψ remains $\frac{1}{\sqrt{2}}(\psi_0 - i\psi_1)$

Then P2 completes transition, $\psi \rightarrow \psi_1$

and M2 gives result ψ_1 every time

- If M1 is a measurement, then ψ is either ψ_0 or ψ_1

If $\psi = \psi_0$, then P_2 drives $\psi \rightarrow \frac{1}{\sqrt{2}}(\psi_0 - i\psi_1)$

If $\psi = \psi_1$, then P_2 drives $\psi \rightarrow \frac{1}{\sqrt{2}}(\psi_1 - i\psi_0)$

Either way, M_2 gives ψ_0 or ψ_1 , both with 50% probability.

So repeat sequence many times, and see what M_2 gives.

First experiment: Make M_1 as described above

Then see M_2 vary, as expected.

Second experiment: Make M_1 as described, but don't watch detector.

See M_2 vary. Seems to rule out A.

Third experiment: Apply laser pulse during M_1 , but remove detector entirely.

See M_2 vary. Seems to rule out B.

Fourth experiment: Don't apply laser during M_1 .

See M_2 gives ψ_1 every time, as expected.

So this would seem to mean that C is the answer.
But then why doesn't Schr. eqn work here?

Compare, if we don't apply laser, then field state remains ϕ_0 .

$$\begin{aligned} \text{After P2, have } \Psi &= \frac{1}{\sqrt{2}} \left[\frac{1}{\sqrt{2}} (\alpha_0 - i\alpha_1) - i \frac{1}{\sqrt{2}} (\alpha_1 - i\alpha_0) \right] \phi_0 \\ &= \frac{1}{2} [\alpha_0 - i\alpha_1 - i\alpha_1 - \alpha_0] \phi_0 \\ &= -i\alpha_1 \phi_0 \end{aligned}$$

Get α_1 every time, as 4th experiment showed.

So it turns out that "measurements" can be explained without collapsing wave function after all.

Important part is really that state of atom 2 becomes correlated with state of field ϕ .

Word for this kind of quantum correlation is "entanglement".

We'll talk more about entanglement in next few days.