Lecture 35

Recap:
Spontaneous emission: excited states have finite lifetimes, eventually make spontaneous transition to ground state.

Adiabatic approximation: if $H$ changes slowly enough, system stays in same state while state itself changes.

Rest of the course:
No more hard calculations, say something about what QM means.

Basic question: what exactly is a wavefunction? Can't observe it directly; behaves in some strange ways.

Today: focus on one strange thing: measurement.

Suppose we measure something about a particle in state $\Psi$. To be concrete, let's measure energy.

QM says: to predict results, first write $\Psi$ as sum of eigenstates of energy operator ($H$):

$$\Psi = \sum_n c_n \Psi_n$$

Then measurement gives result $E_n$ with prob $|c_n|^2$.

Also, after measurement, system is in state $\Psi_n$.

Say that wavefunction "collapses" to $\Psi_n$.

Clear enough, and agrees well with experiments.
But it is kind of strange.

Evolution of $\Psi$ normally governed by Schröd. Eqn.,
but Schröd. Eqn. doesn't have "collapse" in it.

- Seems inelegant to need two separate types of evolution
- Also, what constitutes a measurement? When exactly do we need to impose this "collapse"?

To get a better idea, consider a measurement example.

Recall hydrogen atom has two ground states due to hyperfine effect.

\[
\begin{align*}
1S & \quad F = 1 \\
& \quad \text{Upper state does decay via spontaneous emission, but lifetime is } \approx 4000 \text{ years.}
\end{align*}
\]

Can prepare an atom in a superposition of $|F_0 = 0\rangle \pm |F_1 = 1\rangle$
by applying B-field oscillating at 1.46 Hz

(Need B, not E, because we're affecting spin instead of charge.)

If we start at $|F_0\rangle$, and apply resonant field $\Delta = 0$
we have
\[
C_1(t) = -i \sin \frac{\Delta t}{\hbar} \quad 5.2 \times 10^{-7}
\]

In particular, apply for time $t = \frac{\pi}{2} \cdot \frac{1}{4}$

After pulse, $\Psi(t) = \frac{1}{\sqrt{2}} (|F_0\rangle - i |F_1\rangle)$
Now, we measure state by applying laser tuned between F = 1 state and 2p excited state.

Set laser strength & duration to drive transition with 100% probability.

Then wait several ns. If atom in 2p state, it will decay, emitting a photon.

So if we detect photon, know atom was in F = 1 state. Furthermore, if we make laser with correct polarization, we can be sure 2p state decayed back to F = 1 state.

So if we detect photon, know atom is in F1, if no photon, know atom in F0.

This is one measurement.

Question is, what makes it a measurement?

Some possible answers:

A: Fact that I, a conscious being, learned the state of the system. Important part is the observer.

B: Fact that the state of my detector changed depending on the state of the atom. Important part is interaction with a macroscopic object.

C: Fact that atom scattered a photon at all. Important part is interaction with anything.
All of these have some merit, but also some problems:

A: Does that mean the universe would behave totally differently if humans weren't around?

B: How big does something have to be to count as macroscopic?

C: But photons are quantum entities. Shouldn't the Schrödinger equation handle this by itself?

We can imagine an experiment to resolve these options.

Sequence:
I. Prepare atom in $|\psi\rangle$
P1. Apply B-field pulse $\Delta t \approx \frac{1}{\hbar} (\frac{1}{2} - i \frac{1}{4})$
M1. Attempt measurement in various ways
P2. Apply B-field again
M2. Measure again in straightforward way

Two possibilities for M2:
- If M1 is not a successful measurement, then $\psi$ remains $\frac{1}{2} (|\psi_0\rangle - i |\psi_1\rangle)$

Then P2 completes transition, $\psi \rightarrow \psi_1$
and M2 gives result $\psi_1$ every time

- If M1 is a measurement, then $\psi_1$ is either $|\psi_0\rangle$ or $|\psi_1\rangle$
If \( y = y_0 \), then \( P_2 \) drives \( z = \frac{1}{2} (y_0 - i \cdot \theta_0) \).

If \( y = \theta_1 \), then \( P_2 \) drives \( z = \frac{1}{\sqrt{2}} (y_1 - i \cdot \theta_0) \).

Either way, \( M_2 \) gives \( z = \theta_1 \), both with 50% probability.

So repeat sequence many times, and see what \( M_2 \) gives.

**First experiment**: Make \( M_4 \) as described above.

Then see \( M_2 \) vary, as expected.

**Second experiment**: Make \( M_4 \) as described, but don't watch detector.

See \( M_2 \) vary, seems to rule out \( A \).

**Third experiment**: Apply laser pulse during \( M_2 \), but remove detector entirely.

See \( M_2 \) vary, seems to rule out \( B \).

**Fourth experiment**: Don't apply laser during \( M_4 \).

See \( M_2 \) gives \( \theta_1 \), every time, as expected.

So this would seem to mean that \( C \) is the answer.

But then why doesn't \( S_m \) work here?
Compare, if we don’t apply lasers. Then field state remains \( \phi_0 \).

After \( P_2 \), have
\[
\psi = \frac{1}{\sqrt{2}} \left[ \frac{1}{\sqrt{2}} (\psi_0 - i\psi_1) - i \frac{1}{\sqrt{2}} (\psi_1 - i\psi_0) \right] \phi_0
\]
\[
= \frac{1}{\sqrt{2}} \left[ \psi_0 - i\psi_1 - i\psi_1 - \psi_0 \right] \phi_0
\]
\[
= -i\psi_1 \phi_0
\]

Get \( \psi_1 \) every time, as 4th experiment showed.

So it turns out that “measurements” can be explained without collapsing wave function after all.

Important part is really that state of atom \( \psi \) becomes correlated with state of field \( \phi \).

Word for this kind of quantum correlation is “entanglement.”

We’ll talk more about entanglement in next few days.