- Final -

Lecture 40

Last time, gave you my interpretation of quantum measurements

If system in state $\psi = \frac{1}{\sqrt{2}}(\psi_0 + \psi_1)$

couple to detector, with wavefunction ϕ, get

$$\psi \to \frac{1}{\sqrt{2}}(\psi_0 \phi_0 + \psi_1 \phi_1)$$

No longer acts like superposition ... ψ_0 & ψ_1 can't interfere
But wavefunction never collapses either

This doesn't resolve all issues, however

- detector might be macroscopic
  Then dealing with wave function of macro object
  ... Schrodinger's cat

or might be a person, or even whole universe!

Does the "wave function of the universe" really make sense?
And if it does, then every time anything happens, universe wave function splits into larger superposition

$$\frac{1}{\sqrt{2}}(\psi_0 + \psi_1)\mathcal{U} \to \frac{1}{\sqrt{2}}(\psi_0 \mathcal{U}_0 + \psi_1 \mathcal{U}_1)$$

Every possible thing that could happen does, in some sense.

Sometimes called "many worlds" interpretation

- Another issue: what if $\psi = \frac{1}{2}\psi_0 + \sqrt{\frac{3}{4}}\psi_1$

Experimentally, observe state 0 25% of time

But if measurement means $\psi \to \frac{1}{2}\psi_0 \phi_0 + \sqrt{\frac{3}{4}}\psi_1 \phi_1$

where does notion of probability come in?

No clear reason to relate amplitudes to frequency of occurence.

- Finally, one problem that all interpretations share evolution of ψ is non local

To sharpen this, consider two atoms A & B States ψ_0, ψ_1

in two-body state $\psi = \frac{1}{\sqrt{2}}\left[\psi_0(A)\psi_1(B) + \psi_1(A)\psi_0(B)\right]$

Prepare state, then put atom A on rocket for α-centauri

Once it gets there, measure particle A

In many worlds idea,

$$\psi \to \frac{1}{\sqrt{2}}\left[\psi_0(A)\psi_1(B)\phi_0 + \psi_1(A)\psi_0(B)\phi_1\right]$$

note whole wave function changes the instant measurement is made
- even though ψ extends across light-years.

Would seem to violate everything about special relativity.

This typically bothers people.

Get same problem in conventional interpretation
Wave function collapses instantly upon measurement.

Spend rest of today on this.

First, a good thing: don't violate causality
Recall that in moving frame, simultaneity can change.

To moving observer, ψ could collapse at α-Centauri
before we measured A on earth.

Seems bad, but of course you can't see ψ collapse.

Imagine trying to send a message faster than speed of light
On earth, Alice measures A
Far away, Bob has B.

If Alice gets ψ_0, she knows Bob has ψ_1
Get that info faster than light, but its not
a message. No one can control whether
she'll get ψ_0 or ψ_1.

Further, Bob's wave function has now collapsed,
but he has no way of knowing this.
If he makes measurement, he'll get 50/50 results,
whether Alice measured first or not.

Note, from Bob's point of view, particle A
has already measured particle B (in many worlds
sense)
So he might as well have said his ψ was already
collapsed, he is just ignorant of result.

Nonetheless, if you think ψ is some kind of physical "thing," you'd think it should obey relativity.

Might suggest that "collapse" actually does happen, and propagates at speed c.
Since you can't observe collapse anyway, why not assume this for sake of consistency?

Run into trouble with something called Bell's Theorem

Imagine quantum particles are electrons
measuring spins

As before, have Alice & Bob far apart
Say Alice measures her particle along axis $\vec{a}$
($\hat{x}$, $\hat{y}$, $\hat{z}$, or any combination)
Bob measures along axis $\vec{b}$

For each, call result of spin up = +1
spin down = −1

Repeat measurements many times.
To compare correlations calculate product of A & B results

For instance, might get

Run	Alice	Bob	Product
1	1	−1	−1
2	1	−1	−1
3	−1	−1	1
4	1	−1	−1
5	−1	−1	1
6	−1	−1	1
			average = 0

Define $P(a,b)$ = average of products.

If $P > 0$, A & B tend to get same thing.

If $P(\vec{a},\vec{b}) < 0$, tend to be opposite
$P(\vec{a},\vec{b}) = 0$, no correlation

If initial state is $\psi = \frac{1}{\sqrt{2}}(\uparrow\downarrow - \downarrow\uparrow)$, then get

$P(\vec{a},\vec{a}) = -1$ (perfect anticorrelation)
$P(\vec{a},-\vec{a}) = +1$

Generally, $P(\vec{a},\vec{b}) = -\vec{a}\cdot\vec{b}$ work out for yourself, problem 4.50

Bell's Theorem

Suppose quantum mechanics has it wrong.
Really, two well separated particles do have spins that are individually well defined, just don't know what they are.

Then there must be some "hidden" parameters $\{\lambda\}$ that encode results a measurement will give.

Evidently, we can't measure λ, but if we could, we could predict results of any measurement of particle.

Then there is some function $A(\vec{a},\lambda) = \pm 1$ gives results for measuring along $\vec{a}$
and $B(\vec{b},\lambda) = \pm 1$

Then have $P(\vec{a},\vec{b}) = \int \rho(\lambda)\, A(\vec{a},\lambda)\, B(\vec{b},\lambda)\, d\lambda$

$\rho(\lambda)\, d\lambda =$ prob that parameters have value λ

But QM says $P(a, b) = -\vec{a}\cdot\vec{b}$

Say $\hat{a} = \hat{x}$, $\hat{b} = \hat{y}$, $\hat{c} = 45°$ in between.

Then $P(a,b) = 0$

$P(a,c) = -\frac{1}{\sqrt{2}}$

$P(b,c) = -\frac{1}{\sqrt{2}}$, so

$$|P(a,b) - P(a,c)| = \frac{1}{\sqrt{2}} = 0.7 \not\leq 1 - \frac{1}{\sqrt{2}} = 0.3$$

Experiments agree with QM,
violate Bell's inequality

Even if A + B measurements are made a long distance apart.

So QM is right. Things really are governed by this weird, non-local wavefunction.