Lecture 41

Last time derived Bell's inequality

If we assume that particles have <u>some</u> intrinsic (+local) property that determines outcome of a measurement, can derive inequality relating different measurements, QM violates inequality, as does experiment

So QM is the best we can do for explaining reality, But it's not very good.

Today, look at some ways to take advantage of quantum weirdness.

quantum computing, - quantum communication

Basic idea:

treat two-level system as a carrier of information.

Classical information: bit = 0 or 1

Quantum information: qubit = $\alpha \psi_0 + \beta \psi_1$

Qubits much more complicated

Defined by 2 continuous variables

But a single qubit can still only transmit at best one bit of info

Say Alice sends Bob state $\psi = \alpha \psi_0 + \beta \psi_1$

Bob can't determine α & β very well, ψ disturbed if he measures it.

Alice best of sending either ψ_0 or ψ_1
Then at least Bob knows what he's gotten

So why use qubits?
Because N qubits can do more than N bits

Show a couple examples.

Quantum dense coding:

Alice can send 2 bits to Bob with one qubit

<u>if</u> they have previously exchanged some entangled particles

Suppose Alice starts with two atoms in state

$$\psi = \frac{1}{\sqrt{2}}\left[\psi_0(A)\psi_1(B) - \psi_1(A)\psi_0(B)\right]$$

She gives atom B to Bob, who goes away on a trip

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Later on, Alice wants to send Bob a message, one of four possible values.

If message = "1", she sends Bob her atom directly

If message = "2", she flips state of her particle

$$\psi_0 \leftrightarrow \psi_1$$

(By applying oscillating B field)

Then sends it to Bob

If message = "3", she applies π-phase shift to state ψ_1

For instance, apply static B-field

Zeeman shift changes $E_1 \to E_1 + \delta$

(Assume E_0 doesn't change)

Apply for time $t = \pi \frac{\hbar}{\delta}$

Then $\psi_1 \to \psi_1 e^{-i\frac{\delta t}{\hbar}} = \psi_1 e^{-i\pi} = -\psi_1$

Then she sends atom to Bob.

If message = "4" she changes state <u>and</u> changes phase

Then sends to Bob.

So what does Bob get?

If 1, $\psi = \frac{1}{\sqrt{2}}\left[\psi_0(A)\psi_1(B) - \psi_1(A)\psi_0(B)\right]$

If 2, $\psi = \frac{1}{\sqrt{2}}\left[\psi_1(A)\psi_1(B) - \psi_0(A)\psi_0(B)\right]$

If 3, $\psi = \frac{1}{\sqrt{2}}\left[\psi_0(A)\psi_1(B) + \psi_1(A)\psi_0(B)\right]$

If 4, $\psi = \frac{1}{\sqrt{2}}\left[\psi_1(A)\psi_1(B) + \psi_0(A)\psi_0(B)\right]$

These are four orthogonal states

Called "Bell basis"

Bob can measure both particles in this basis,

and will get one of 4 definite results

He then knows message

qubit A transmitted 2 bits

Note, Alice and Bob still exchanged two qubits overall. (Atoms A and B)

But they were able to exchange one of them before they knew what the message was!

So you can definitely do some things with qubits that you can't do with bits.

Another fancier example: quantum teleportation

In general, if Bob has a quantum particle in an unknown state $\psi = a\psi_0 + b\psi_1$

the only way to give this state to Alice is to send particle

Can't measure a and b and tell Bob
- Bob would need ∞ # of copies of state to do that.

But suppose Alice & Bob started out with particles in shared state

$$\frac{1}{\sqrt{2}}\left[\psi_0(A)\psi_1(B) + \psi_1(A)\psi_0(B)\right]$$

Later, Bob gets atom C in state $\phi = a\psi_0(c) + b\psi_1(c)$

Then Bob measures atoms A and C in Bell basis

What happens?

Before measurement, three particle state is

$$\psi = \frac{1}{\sqrt{2}}\Big[a\,\psi_0(A)\psi_1(B)\psi_0(C) + a\,\psi_1(A)\psi_0(B)\psi_0(C) + b\,\psi_0(A)\psi_1(B)\psi_1(C) + b\,\psi_1(A)\psi_0(B)\psi_1(C) \Big]$$

By adding & subtracting identical terms, can rewrite as

$$\psi = \frac{1}{2^{3/2}}\Big[aA_1B_0C_0 + aA_1B_1C_1 + bA_0B_0C_0 + bA_0B_1C_1$$
$$+ aA_1B_0C_0 - aA_1B_1C_1 - bA_0B_0C_0 + bA_0B_1C_1$$
$$+ aA_0B_0C_1 + aA_0B_1C_0 + bA_0B_0C_1 + bA_1B_1C_0$$
$$- aA_0B_0C_1 + aA_0B_1C_0 + bA_0B_0C_1 - bA_1B_1C_0 \Big]$$

This factors to

$$\psi = \frac{1}{2^{3/2}}\Big[(aA_1 + bA_0)(B_0C_0 + B_1C_1)$$
$$+ (aA_1 - bA_0)(B_0C_0 - B_1C_1)$$
$$+ (aA_0 + bA_1)(B_0C_1 + B_1C_0)$$
$$- (aA_0 - bA_1)(B_0C_1 - B_1C_0) \Big]$$

Bob's measurement collapses ψ to one of these terms (& he knows which one.)

So after measurement, Alice has one of four states related to ϕ

Bob then sends Alice two classical bits saying which Bell state he got

Then Alice can make local operations (flipping state and/or changing phase) to recover ϕ.

Note again, Alice + Bob did need to exchange a particle, but they were able to do it ahead of time.

Also note, neither Alice nor Bob ever knows what state ϕ is.

Called teleportation because atom A is now in same state that atom C was originally.

Since atoms are indistinguishable, this is physically the same as transporting C itself.

If we could do this with 10^{23} atoms all at once, we could be like Star Trek!

Other fancy tricks you can do. Most complicated is Shor's Algorithm:

If you could make arbitrary manipulations on quantum state of 2N particles,

can factor a number up to 2^N fairly quickly

Important for cryptography applications