Lecture 42

Remember, final at 9:00 am Saturday, in this room
Bring book, notes, calculator, scratch paper

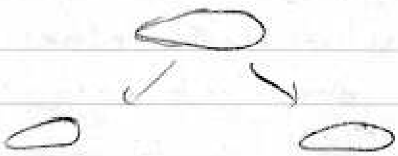
Office hours
- When should I hold them?
- Study early and come!

Course evals:
- Only 4 done so far.
- Open until May 4, 9:00 AM
- Need to do them. If you don't, I won't drop your low HW score.

Last time, talked about some applications of quantum entanglement
- Quantum dense coding
- Quantum teleportation

Today, last bit on my experiment: atom interferometry

Say we take an atomic wave function & split into spatially separate parts:



$$\psi = \frac{1}{\sqrt{2}}(\psi_1 + \psi_2)$$

If atom in a weak potential $V(x)$
then each packet aquires phase

$$\psi \to \frac{1}{\sqrt{2}}\left(\psi_1 e^{-i\frac{V(x_1)t}{\hbar}} + \psi_2 e^{-i\frac{V(x_2)t}{\hbar}} \right)$$

$$\doteq \frac{1}{\sqrt{2}}\left(\psi_1 + e^{-i\frac{\Delta V t}{\hbar}} \psi_2 \right)$$

Can use this to measure very small ΔV's
For instance, if seperation is vertical, have
gravitational potential

$$\Delta V = mgd \qquad d = x_2 - x_1$$

Say $t = 10\,ms$ and $d = 0.1\,mm$. Then

$$\phi = \frac{mgdt}{\hbar} \simeq 1400 \text{ rad}$$

$m = 87u$, Rb atoms

That's pretty big.
If we could measure ϕ precisely, could
use to get very accurate value of g

Gravity info useful for navigation, oil prospecting

But could measure anything with a potential gradient
E&M fields, interactions with other atoms,
accelerations, even rotation.

So this could potential be a useful measurement
technique.

We are trying to implement with BECs
Lots of atoms in same state, makes interference easy to see.

Mostly want to explain details of how atom manipulation works. Nice example of driving transitions

Start with atoms in wave packet at rest : ψ_0
(assume packet big enough that Δp is negligible.)

Apply standing wave laser. $E = E_0 \cos kz \cos \omega t$

But tune ω off resonance... don't drive transitions to excited state.

Do get a second order energy shift, however. Turns out

$$\Delta E = \beta \langle |E|^2 \rangle$$ don't worry about β

So atom sees potential $V = \beta E_0^2 \cos^2 kz$

$$\rightarrow V_0 \cos 2kz$$

while laser is one.

For perturbation that turns on at time 0 and off at time t, have transition prob

$$P_{abs} = \frac{|V_{ba}|^2}{\hbar^2 \omega_r^2} \sin^2 \omega_r t$$

$$\omega_r = \frac{1}{2}\sqrt{\omega_0^2 + 4\frac{|V_{ba}|^2}{\hbar^2}}$$

But what is state b here?

Well, if $V = U_0 \cos 2kz$

Then $\langle \psi | U | \psi_0 \rangle$ is non zero only if ψ looks like $\cos 2kz \, \psi_0$

$\sim e^{i2kz} + e^{-i2kz}$ since $\psi_0 \sim$ constant

These are plane waves $e^{i\frac{pz}{\hbar}}$ moving with momentum $p = \pm 2\hbar k$

Picture:

photons $\rightarrow$ ○ $\leftarrow$ photons

Absorb photon from one beam. Pick up momentum $\hbar k$

Since laser off resonance, doesn't conserve energy.

Stay in excited state only for time $\Delta t \sim \frac{\hbar}{\Delta E} \sim \frac{1}{|\omega - \omega_0|}$

Then other beam drives transition down

emit photon $p = -\hbar k$

atom recoils to $p = +2\hbar k$

Absorb & emit from either beam, so get superposition

$$\psi \to \frac{1}{\sqrt{2}}\left(e^{2ikz} + e^{-2ikz}\right) \equiv \psi_+$$

So, split condensate into two packets travelling apart.

Might complain that this isn't a two-level system. What about $\pm 4\hbar k$ states?

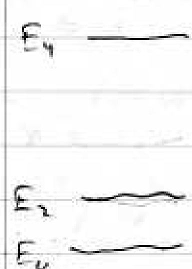
Think about energies

Energy of ψ_0 is $E_0 = 0$

Energy of e^{2ikz} is $\frac{p^2}{2m} = \frac{(2\hbar k)^2}{2m} = 2\frac{\hbar^2 k^2}{m}$

Energy of e^{4ikz} is $\frac{(4\hbar k)^2}{2m} = 8\frac{\hbar^2 k^2}{m}$

Picture



E_0 & E_2 much closer than E_2 & E_4

Since potential is not oscillating, no transition is resonant

$0 \to 2$ is closer, works better
$2 \to 4$ is suppressed, higher ones even more

We do some tricks to make $0 \leftrightarrow 2$ work better anyway.

After atoms fly apart for time T, apply laser again.

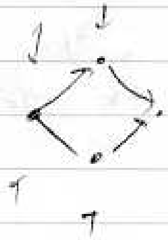
This time, use less intensity, (longer pulse) V is smaller, can't couple ψ_+ to ψ_0 very well due to ΔE.

But, transition $e^{2ikz} \to e^{-2ikz}$ is resonant... states have no ΔE.

Drive this in 2nd order

Effect: atoms reflected

Picture:



When packets cross again, apply splitting pulse just as before.

Know transitions are reversible, so $\psi_+ \to \psi_0$
atoms come back to rest

But what if packets acquired differential phase ϕ due to gravity or something.

Then before recombination, $\psi = \frac{1}{\sqrt{2}}\left(e^{2ikz} + e^{i\phi} e^{-2ikz}\right)$

$$= \frac{1}{\sqrt{2}} e^{i\phi/2}\left(e^{i(2kz-\phi/2)} + e^{-i(2kz-\phi/2)}\right)$$

$$= \sqrt{2}\cos\left(2kz - \phi/2\right)$$

$$= \sqrt{2}\left[\cos 2kz \cos\frac{\phi}{2} + \sin 2kz \sin\frac{\phi}{2}\right]$$

$$= \left[\cos\frac{\phi}{2}\,\psi_+ + \sin\frac{\phi}{2}\,\psi_-\right]$$

where $\psi_- \equiv \frac{1}{\sqrt{2}}\left(e^{2ikz} - e^{-2ikz}\right)$

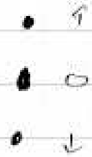
Now ψ_- isn't coupled by $\psi_0 \leftrightarrow \psi_+$ transition
it just stays ψ_-

So after recombination, left with

$$\psi = \cos\frac{\phi}{2}\,\psi_0 + \sin\frac{\phi}{2}\,\psi_-$$

Let atoms propagate longer. Ones in ψ_0 sit still, ones in ψ_- move apart.

Finally, take picture, see three spots:



Fraction of atoms at rest $= \cos^2 \frac{\phi}{2}$

Allows measurement of ϕ, what we wanted.

We've made this work, $T = 11$ ms

packets separated by ~ 0.25 mm

"Macroscopic" superposition state of about 10^4 atoms.