Presentations next week

Final major result of exchange symmetry:
effect on statistical mechanics

Statistical Mechanics:
tools for dealing with system when we don't/can't know what state it is in.

For instance, 10^{23} gas molecules in a box
Impossible to write down quantum state,
wouldn't want to see it if you did

- Instead say you know total energy E [micro canonical]
  Assume that system is equally likely to be in any (many body) quantum state with that E

- Typically lots & lots of many-body states within dE of E

To characterize system, need to count them

Problem 5.23
Say three noninteracting particles in 10 harmonic oscillator, freq ω. Total energy $= \frac{9}{2}\hbar\omega$

- What are possible configurations?
- How many distinct states in each?
- What is most probable configuration?
- What is prob for a given particle to have various energies?
- What is most probable energy?

a) Start with distinguishable particles

Single particle energies $\hbar\omega(n+\frac{1}{2})$ $n=0,1,\ldots$

Three particles, total zero point energy = $\frac{3}{2}\hbar\omega$

leaves $3\hbar\omega$ available

Could have three configurations: How many particles in each state

I. one with $n=3$, two with $n=0$

II, one with $n=2$, one with $n=1$, one with $n=0$

III; three with $n=1$

Three ways to achieve I, $(3,0,0)$, $(0,3,0)$, $(0,0,3)$

Six ways to achieve II, $(2,1,0)$ $(1,2,0)$, $(2,0,1)$ $(0,2,1)$ $(1,0,2)$ $(0,1,2)$

One way to achieve III, $(1,1,1)$

= 10 possible states

↳ Labeling of which particle in which state

Each state equally likely:

II is most probable (60% chance)

If I select a particle at random, what is chance to have $n=3$?

Need to be in configuration I, and need to select right particle.

$$P_3 = \frac{3}{10} \times \frac{1}{3} = \frac{1}{10}$$

$$P_2 = \frac{6}{10} \times \frac{1}{3} = \frac{2}{10}$$

$$P_1 = \frac{6}{10} \times \frac{1}{3} + \frac{1}{10} \times \frac{3}{3} = \frac{3}{10}$$

$$P_0 = \frac{3}{10} \times \frac{2}{3} + \frac{6}{10} \times \frac{1}{3} = \frac{4}{10}$$ ← Most probable energy

b) Identical bosons

Same configurations as before

I: One with $n=3$, two with $n=0$

II: One with $n=2$, one with $n=1$, one with $n=0$

III: Three with $n=1$

But since bosons indistinguishable, don't have distinct states within each configuration

→ Wave function is superposition of all the states:

$$\psi_I(A,B,C) = \frac{1}{\sqrt{3}}\left[\psi_3(A)\psi_0(B)\psi_0(C) + \psi_0(A)\psi_3(B)\psi_0(C) + \psi_0(A)\psi_0(B)\psi_3(C)\right]$$

etc

So only three different states, total

Each configuration is equally probable, $P = \frac{1}{3}$

Select a particle at random...

$P_3 = \frac{1}{3} \times \frac{1}{3} = \frac{1}{9}$

$P_2 = \frac{1}{3} \cdot \frac{1}{3} = \frac{1}{9}$

$P_1 = \frac{1}{3} \times \frac{1}{3} + \frac{1}{3} \cdot \frac{3}{3} = \frac{4}{9}$ ← most probable

$P_0 = \frac{1}{3} \times \frac{2}{3} + \frac{1}{3} \times \frac{1}{3} = \frac{3}{9}$

c) Identical fermions (ignoring spin)

Can't have more than one particle in same single-particle state

Can't have I or III

Only II, and only one state for this configuration.

Then have

$P_3 = 0 \quad P_2 = \frac{1}{3} \quad P_1 = \frac{1}{3} \quad P_0 = \frac{1}{3}$

$n = 0, 1,$ or 2 equally likely

That's an example of the kind of reasoning, but stat mech is about doing it in general, not just for specific cases.

General problem:

N particles in potential with single-particle energies $E_1, E_2, \ldots$

In 3D problems, usually have several single-particle states all with same energy

For instance, 3D harmonic oscillator

$E_1 = \frac{3}{2}\hbar\omega$ one state $(n_x, n_y, n_z) = (0,0,0)$

$E_2 = \frac{5}{2}\hbar\omega$ three states $(1,0,0)$ $(0,1,0)$ $(0,0,1)$

$E_3 = \frac{7}{2}\hbar\omega$ six states $(2,0,0)$ $(0,2,0)$ $(0,0,2)$ $(1\,1\,0)$ $(1\,0\,1)$ $(0\,1\,1)$