Lecture 9

Wrap up stat mech

Last time, derived most probable occupation numbers for system

N_n = # of particles with energy E_n

$$= e^{-(\alpha + \beta E_n)} \qquad \text{disting.}$$

$$= \frac{1}{e^{\alpha+\beta E_n}+1} \qquad \text{fermions}$$

$$= \frac{1}{e^{\alpha+\beta E_n}-1} \qquad \text{bosons}$$

Choose α & β to get correct total N and E

$$\sum_{n=1}^{\infty} N_n = N$$

$$\sum_{n=1}^{\infty} N_n E_n = E$$

Example:
Particles in a box

For disting. particles, got

$$e^{-\alpha} = \frac{N}{V}\left(\frac{2\pi\hbar^2\beta}{m}\right)^{3/2}$$

and $\beta = \frac{3}{2}\frac{N}{E}$

But wait: noninteracting particles in a box is an ideal gas.

From thermodynamics, know energy per particle in ideal gas:

$$\frac{E}{N} = \frac{3}{2}k_B T = \frac{3}{2}\frac{1}{\beta}$$

Evidently $\beta = \frac{1}{k_B T}$

Note this is totally different from thermodynamics approach
- ties together microscopic & macroscopic models

Also get $\alpha = -\frac{\mu}{k_B T}$ μ = chemical potential

Unfortunately, can't do integrals for bosons & fermions

$$N = \int_0^\infty \frac{1}{\exp\left[\left(\frac{\hbar^2 k^2}{2m} - \mu\right)/kT\right] \pm 1} \frac{V}{2\pi^2} k^2 dk$$

But in limit $E_k - \mu > kT$, exp is large and looks like distinguishable particles

In that limit, $\mu = -\alpha kT = kT \ln\left[\frac{N}{V}\left(\frac{2\pi\hbar^2}{mkT}\right)^{3/2}\right]$ p 240

Call $\sqrt{\frac{mkT}{2\pi\hbar^2}} = \Lambda$,
Thermal deBroglie wavelength
Typical wavelength for particles with energy $\sim kT$

So $\mu = kT \ln[\rho \Lambda^3]$

ρ = density $= \frac{N}{V}$

$\rho\Lambda^3$ = # of particles w/in a cubic wavelength

For typical gas $\rho \Lambda^3 \ll 1$ $[\rho \sim 10^{25} m^{-3} \quad \Lambda \sim 10^{-10} m]$

so $\ln[\rho \Lambda^3] < 0$

$\mu \sim -kT$

So can ignore exchange effect if

$E_k - \mu > kT$

$\rightarrow E_k + kT > kT$

$\rightarrow E_k > 0$, true for all states!

Rule of thumb:

Can use distinguishable particle function if $\rho \Lambda^3 \ll 1$

otherwise, need to calc boson or fermion functions numerically.

Already know that in solids, need to use fermion function.

Now we could find state at nonzero T too.

Lots more to say, but I'll let you see it in Stat Mech.

Wraps up Chapter 5

Start Chapter 6 - Perturbation Theory

Go back to thinking about one or maybe two particle systems.

Basic problem:

Can only solve Schr. Equation for

$U = 0$ (free particle, particle in box, δ-fcn)

r^2 (harmonic oscillator)

$\frac{1}{r}$ (Hydrogen atom)

Those are all pretty important, but lots of other U's come up.

One solution: solve numerically
- Doesn't give a lot of insight
- Rapidly becomes hard for more than one particle

Sometimes another possibility:

Say Hamiltonian $H = H^0 + H'$

H^0 = "simple" Hamiltonian that we know how to solve

H' = deviation from H^0 = perturbation

For instance, anharmonic oscillator

$$H = \frac{p^2}{2m} + \frac{1}{2} m\omega^2 x^2 + \alpha x^3$$

$$H' = \alpha x^3$$

Method: assume effect of H' is fairly small, calculate changes starting from H^0

For convenience, consider

$$H = H^0 + \lambda H' \quad , \quad \lambda \ll 1$$

If H' is itself small, we can eventually take $\lambda \to 1$

Use λ for now because we don't know how to quantify smallness of H'

Intuitive expect eigenstates of H to have form

$$\psi_n = \psi_n^0 + \lambda \psi_n^1 + \lambda^2 \psi_n^2 + \ldots$$

$$E_n = E_n^0 + \lambda E_n^1 + \lambda^2 E_n^2 + \ldots$$

where ψ_n^0 is eigenstate of H^0 with energy E_n^0

ψ_n^1, E_n^1 $\equiv$ first order correction, etc

Schrodinger says $H \psi_n = E_n \psi_n$

$$(H^0 + \lambda H')(\psi_n^0 + \lambda \psi_n^1 + \lambda^2 \psi_n^2 + \ldots)$$

$$= (E_n^0 + \lambda E_n^1 + \lambda^2 E_n^2 + \ldots)(\psi_n^0 + \lambda \psi_n^1 + \ldots)$$