Phys 356 Midterm Exam Due March 3

Instructions:

This is a take home, unlimited time exam. You may refer to your textbook, class notes, and homework solutions. You may also use a book or computer program to look up integrals, but you should note what reference you are using and when. You may not use any other references besides these, and you may not discuss the exam with other people. You may use a calculator, but you shouldn't expect to need one.

The exam consists of four problems, each worth 10 points. For full credit, you must explain your reasoning and show all your work in detail. It is always a good idea to neatly rewrite your final solutions before turning them in.

The exam is due at the begining of class on Friday, March 3. Late exams will not be accepted without prior approval. Turn in your final solutions stapled to these exam sheets.

If you have any questions about the exam, don't hesitate to contact me by email (sackett@virginia.edu), phone (924-6795), or coming to my office. Should any of the problems need clarification, I will send an email to the whole class and post the revision on the class website.

Name: ________________________________

Pledge: ____

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1. Consider two spin-1/2 electrons in an infinite 3D square well, with

$$V(\mathbf{r}) = \begin{cases} 0 & \text{if } 0 < x < a,\ 0 < y < a, \text{ and } 0 < z < a \\ \infty & \text{otherwise.} \end{cases}$$

Neglect any interaction between the particles.

(a) Determine the degeneracy of the ground state, and write down a normalized manybody wave function for each state you find. Note that you should include the spin degree of freedom.

(b) Perform the same analysis for the first excited state(s).

When writing down the wave functions, you don't need to write out the explicit functional form. In other words, feel free to define the single-particle states ψ_n and write the many body state in terms of the ψ_n.

2. Consider a system consisting of just two single particle states, ψ_A and ψ_B, with energies E_A and E_B. If the system is occupied by six particles, determine the different possible configurations and the number of distinct many-body states corresponding to each configuration, assuming:

(a) that the particles are distinguishable

(b) that the particles are bosons.

For each case, if each many-body state is equally likely to be occupied, what is the probability that all six particles will be found to have the same energy?

3. Consider two particles in a one-dimensional infinite square well of size a. The particles interact through a potential

$$V(x_1, x_2) = Ua\delta(x_1 - x_2).$$

You can ignore the particles' spin, or equivalently just assume that both particles are in the same spin state. Use first order perturbation theory to calculate the energies of the ground state and the first excited states assuming the particles are:

(a) fermions

(b) bosons

(c) distinguishable.

Be sure to include all degenerate states in your calculations.

4. Calculate the first-order energy shifts of the various $n = 2$ states of the hydrogen atom in response to the perturbation

$$H' = \beta \cos\theta \, \cos\phi,$$

where θ and ϕ are the usual polar and azimuthal angles of spherical cooridinates. Ignore spin and fine structure effects for this problem. *Hint:* Don't do any hard integrals until you are sure that you need them. They might be multiplied by a simpler integral which is zero.

Note that this is not a physically reasonable perturbation, since it is indefinite on the the z-axis. However, it works for the problem at hand.