Supplement 1
An exchange symmetry problem:

Suppose two single particle states

$$\psi_A(x) = \left(\frac{2}{\pi}\right)^{1/4} e^{-(x-a)^2} \quad : \text{Gaussian centered at } +a$$

$$\psi_B(x) = \left(\frac{2}{\pi}\right)^{1/4} e^{-(x+a)^2} \quad : \text{Gaussian centered at } -a$$

These are normalized but not orthogonal.

Let's see what symmetrized wave fcn looks like

Have

$$\psi_\pm(x_1,x_2) = A\left[\psi_A(x_1)\psi_B(x_2) \pm \psi_B(x_1)\psi_A(x_2)\right]$$

$$= A\sqrt{\frac{2}{\pi}}\, e^{-(x_1^2+x_2^2+2a^2)}\left[e^{2a(x_1-x_2)} \pm e^{-2a(x_1-x_2)}\right]$$

(after a little algebra)

Easiest to use center of mass coordinates

$$X = \frac{x_1+x_2}{2} \qquad x = x_1 - x_2$$

Then $x_1 = X + \frac{x}{2}$ $\qquad x_2 = X - \frac{x}{2}$

$$x_1^2 + x_2^2 = 2X^2 + \frac{x^2}{2}$$

$$\boxed{\psi_\pm(X,x) = A\sqrt{\frac{2}{\pi}}\, e^{-\left(2X^2+\frac{x^2}{2}+2a^2\right)}\left[e^{2ax} \pm e^{-2ax}\right]}$$

Let's normalize this, determine A

$$|\psi|^2 = A^2 \frac{2}{\pi} e^{-4a^2} e^{-4X^2} e^{-x^2}\left[e^{4xa} + e^{-4xa} \pm 2\right]$$

Need $\int |\psi|^2 \, dX \, dx = 1$

$$\int e^{-4X^2} dX = \frac{\sqrt{\pi}}{2}, \quad \text{so}$$

$$\int |\psi|^2 dX = A^2 \frac{1}{\sqrt{\pi}} e^{-4a^2} e^{-x^2}\left[e^{4xa} + e^{-4xa} \pm 2\right]$$

Have $\int e^{-x^2} dx = \sqrt{\pi}$

$$\int e^{-x^2} e^{\pm 4xa} dx = e^{4a^2} \int e^{-(x \mp 2a)^2} dx = \sqrt{\pi}\, e^{4a^2}$$

So $\int |\psi|^2 dX \, dx = A^2 e^{-4a^2}\left(2e^{4a^2} \pm 2\right) = 1$

$$A^2 = \frac{1}{2} \frac{1}{1 \pm e^{-4a^2}}$$

$$\boxed{A = \frac{1}{\sqrt{2}} \frac{1}{\sqrt{1 \pm e^{-4a^2}}}}$$

So if $a \to \infty$, states are nearly orthogonal,
get $A \to \frac{1}{\sqrt{2}}$ as in prob. 5.4(a)

If $a \to 0$, states are the same, get
$A \to \frac{1}{2}$ for bosons
$\to \infty$ for fermions
As in prob. 5.4(b)

Have total

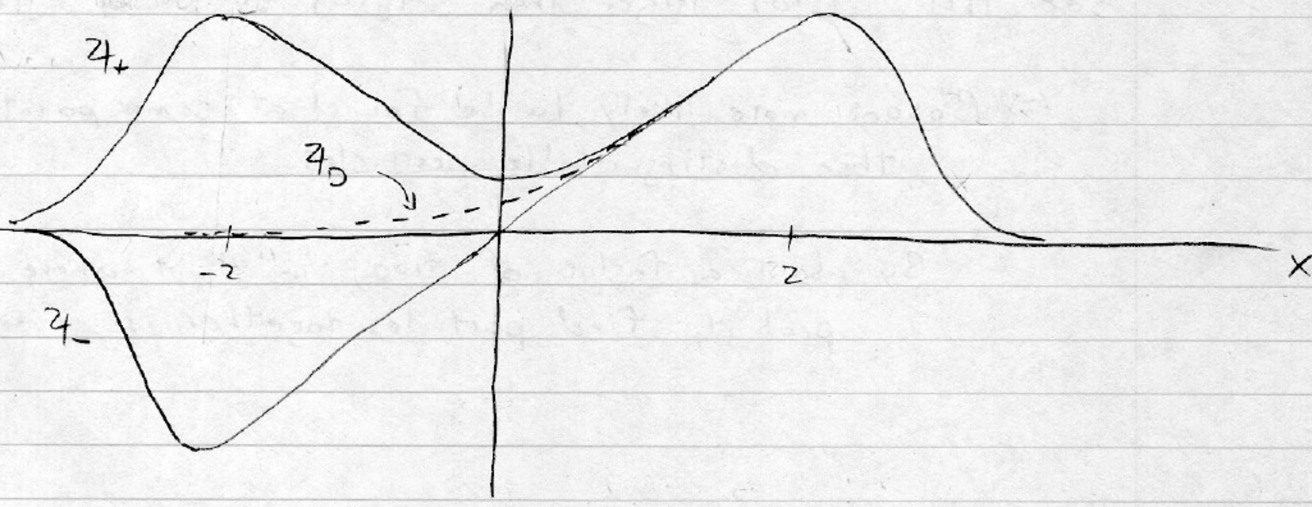
$$\psi_\pm(X,x) = \frac{1}{\sqrt{\pi}} \frac{1}{\sqrt{1 \pm e^{-4a^2}}} e^{-(2X^2 + \frac{x^2}{2} + 2a^2)} \left[e^{2ax} \pm e^{-2ax} \right]$$

$$= \psi_{cm}(X)\, \psi_{rel}(x)$$

$$\psi_{cm}(X) = \left(\frac{4}{\pi}\right)^{1/4} e^{-2X^2} \qquad \text{Simple Gaussian}$$

$$\psi_{rel}(x) = \left(\frac{1}{4\pi}\right)^{1/4} \frac{e^{-2a^2}}{\sqrt{1 \pm e^{-4a^2}}} \left[e^{2ax} \pm e^{-2ax} \right] e^{-\frac{x^2}{2}}$$

Sketch ψ_{rel}, for $a=1$



Compare to $\psi_D(x_1, x_2) = \sqrt{\frac{2}{\pi}}\, e^{-(x_1 - a)^2}\, e^{-(x_2 + a)^2}$

$$= \left(\frac{4}{\pi}\right)^{1/4} e^{-2X^2} \cdot \left(\frac{1}{\pi}\right)^{1/4} e^{-\frac{1}{2}(x - 2a)^2}$$

(peaked at $x = 2a$)

= amp for particles to be at same position

Finally, compare $\psi(x=0)$ for various cases.

$\psi_-(0) = 0$ always

$\Rightarrow$ two fermions can't be at the same point in space

$$\psi_D(0) = \left(\frac{1}{\pi}\right)^{1/4} e^{-2a^2}$$

$$\psi_+(0) = \left(\frac{1}{\pi}\right)^{1/4} \frac{e^{-2a^2}}{\sqrt{1+e^{-4a^2}}} \cdot 2$$

See that $\psi_+(0)$ larger than $\psi_D(0)$ by factor $\frac{2}{\sqrt{1+e^{-4a^2}}}$

$\Rightarrow$ Bosons more likely to be found at same point than distinguishable particles

By about a factor of two, in limit where prob to find particles together is anyway low.