Homework # 11

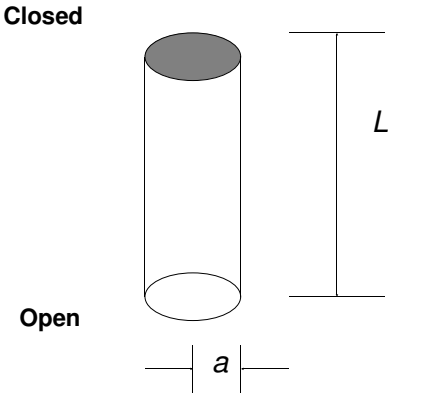
Due: 11/19/97

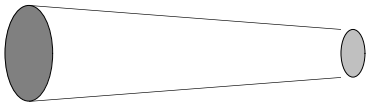
1. Flutes and recorders can be approximated as long cylinders of uniform cross-section, with one end closed and the other open.

   Obviously the normal component of the velocity field (associated with the displacement of air) in the instrument must vanish at the sides and at the closed end. On the other hand, *pressure* fluctuations must vanish at the open end since that is held at constant (atmospheric) pressure.

   Solve the wave equation subject to the above (mixed) boundary conditions, and thereby determine the lowest (fundamental) mode.

   Where would you locate the stops (holes in the side of the instrument) to produce the notes of the 12-tone scale?





2. Show that if two drumheads have the same shape, but one is uniformly scaled with respect to the other by some factor, that the fundamental mode of the smaller drumhead is higher in pitch than the corresponding mode of the larger, independent of the shape.

   (Hint: use the Rayleigh-Ritz variational principle.)

3. The Ruritanian zither is a one-stringed instrument whose string tapers uniformly from one end to the other.

   Estimate the fundamental frequency of a zither of length 2 m, whose string is made of silver (specific gravity 10.6) under tension of 50 Nt, and whose end diameters are 1 and 2 mm, respectively.

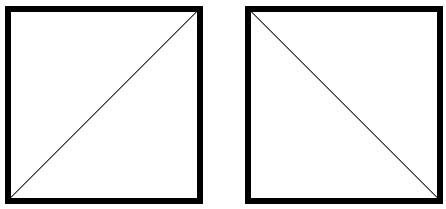
4. A square drumhead of side 1 obviously has (degenerate) normal modes

   $$\psi_1(x,y) = \sin(\pi x)\sin(2\pi y) - \sin(2\pi x)\sin(\pi y)$$

   and

   $$\psi_2(x,y) = \sin(\pi x)\sin(2\pi y) + \sin(2\pi x)\sin(\pi y)$$

   that are related by a reflection.



   Describe qualitatively what happens if a small mass δm is attached at the point $\begin{pmatrix} x=1/4 \\ y=1/4 \end{pmatrix}$.

(over)

Estimate the effect of this mass on the vibration frequencies corresponding to these modes.

Hint: First write down the equation of motion of drumhead plus small mass. Treat the mass density as a function of position:

$$\mu(x,y) = \mu_0 + \delta m\, \delta(x-\tfrac{1}{4})\, \delta(y-\tfrac{1}{4})$$

where $\delta(x)$ is a delta function. The δm term may be regarded as a perturbation to the operator

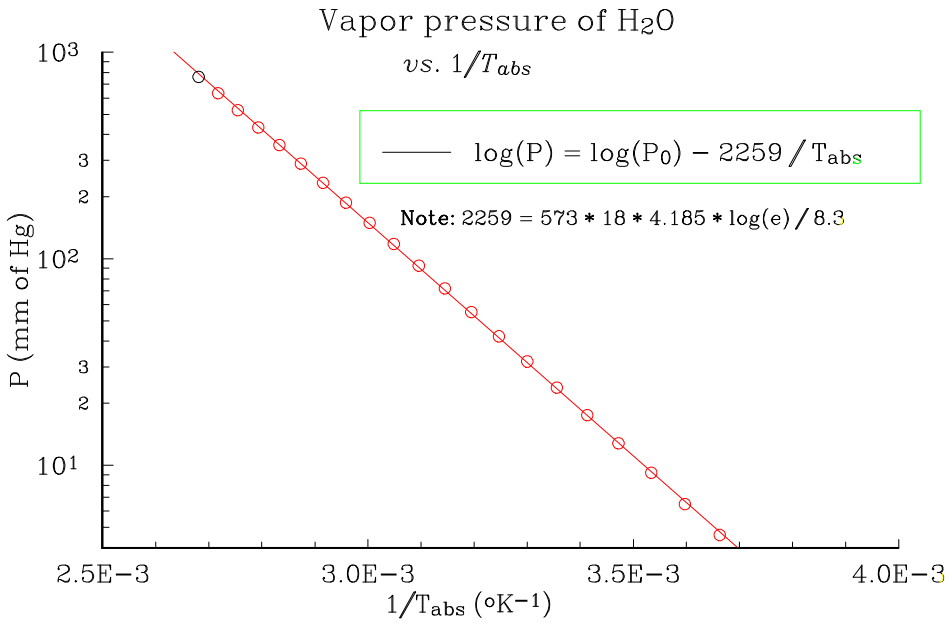
$$-\nabla^2$$

and determine its value to first order in δm.

5. The partial pressure of water vapor at 30 °C is about 32 mm of Hg. (See below for P_{vap} *vs.* T_{abs}.)

a) What is the latent heat of the water vapor in one cm^3 of air, at 100% humidity, at 30 °C (in Joules, please, relative to crystallization into ice)?

b) Estimate the heat energy available in a cylindrical air mass of radius 100 km and height 1 km.

c) What is the maximum fraction of this energy that can be converted to kinetic energy?

d) A 1 MT (megaton) (thermo)nuclear explosive releases 1.2×10^9 kw–hr of energy. Express the kinetic energy of the storm system just described in MT. Estimate the average air speed.