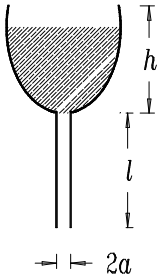
Homework #12

1. What is the velocity of propagation of small-amplitude surface waves on a body of inviscid fluid that has depth h, but is infinite in either of the horizontal directions?

2. A spherical hole of radius a is suddenly formed in an infinite body of incompressible inviscid fluid (not in a graviational field). Assume the pressure at $r = \infty$ is $p(\infty) =$ constant. How long does it take for the hole to disappear?

3. Consider a cylindrically symmetric object (wineglass) of arbitrary shape, $r = R(z)$ and height h, connected as shown to a concentric tube of constant radius a and length l. The glass is initially filled to the top with a fluid of viscosity η and density ρ. At this moment, whatever is keeping it from flowing through the tube is removed. How long does it take for the glass to empty? (Assume that $R(0) = 0$.)



4. Consider the nonstatic case of the one-dimensional atomic model (masses connected by springs) above. Write the equation of motion for the n'th atom's displacement from equilibrium, $\xi_n(t)$.

   Solve the equation of motion and determine the dispersion relation for travelling waves. What is the maximum frequency the system can sustain?

5. Prove that a transformation of the basis,

   $$\hat{e'} = A\,\hat{e}$$

   that diagonalizes a real-symmetric matrix M, is orthogonal, *i.e.*

   $$\tilde{A}\,A = I$$

   where $\tilde{A}$ is the transpose of A.

6. A board (thin beam) of thickness t and width w is clamped at one end (at the top of a wall) and extends a length l beyond the wall. How much does the free end deflect under its own weight?

   If the wood is white pine, 0.75 inches thick and 12 feet long (specific gravity 0.46, Young's modulus 8×10^{10} dyne/cm^2) what is the actual deflection?