Homework #13

1. A beam (not necessarily thin) is clamped at both ends. It extends horizontally in the z–direction, and gravity acts to deflect it downwards ($-x$-direction).

   a) Find the deflection $\xi(z)$ under its own weight.

   b) A weight W is hung from the center. What is $\xi(z)$ now?

2. A horizontal beam is supported at both ends, but the ends are not clamped. If a weight W is hung from the middle, what is $\xi(z)$?

3. A vertical (cylindrical) pillar supports a weight W that is distributed uniformly on its upper surface. Write the equations for compressive deflection as well as any deflection in the horizontal direction.

   a) What is the maximum weight that can be sustained before the pillar becomes unstable?

   b) Suppose the pillar is a hollow cylinder with thin walls ($t = r_2 - r_1 << r_2$). What is the maximum weight now?

4. A thin cylindrical shaft of radius R and length L rotates at angular frequency ω.

   a) What is the maximum value of ω before the shaft becomes unstable with respect to bowing (that is, deflection perpendicular to the shaft)?

   b) What would be the answer to a) for a thin-walled cylindrical shaft?

5. Water falls from a reservoir located a height h above a turbine and exits the turbine at speed v. What speed corresponds to producing the maximum power from the turbine?