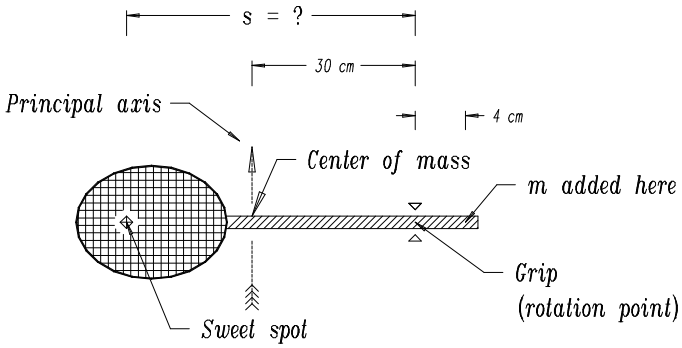
PHYS 521 Name: 15 October 1997
Midterm Examination SS#

Do all 3 problems. Time limit: 1 hr. Open book.

1. A tennis raquet has a mass of 1 kg, is 70 cm long and has a principal moment of inertia (about an axis in the plane of the raquet and perpendicular to the handle) of 0.06 kg-m^2. The present "sweet spot" is located somewhere in the face. The center of mass is located about 30 cm from the grip.



   a) Where is the sweet spot located relative to the grip?

   b) The raquet's owner would like to move the sweet spot about 2 cm further from the grip by wrapping lead (Pb) strip around the grip, thereby adding mass to the grip end of the raquet. How much mass must he add, assuming it is all added 4 cm behind the point of rotation?

Solution

As found in class, the distance of the center of percussion ("sweet spot") from the point of rotation is

$$s = \frac{I_{tot}}{Mx}$$

where x is the distance from center of mass to point of rotation. Now, by the parallel axis theorem, the moment of inertia of the raquet about the point of rotation is

$$I_{tot} = I_{yy} + Mx^2$$

where I_{yy} is the principal moment about the y axis (in the plane of the raquet face).

Thus, the answer to part a) is

$$s = \frac{I_{yy}}{Mx} + x = \frac{0.06\ \text{kg–m}^2}{1\ \text{kg}\ \times 0.3\ \text{m}} + 0.3\ \text{m} = 50\ \text{cm}\ .$$

To find the answer to part b), we note that if we measure from the point of rotation, the position of the new center of mass is

$$x_{new} = \frac{Mx - ma}{M+m}\ ,\ \text{where}\ a = 4\ \text{cm}.$$

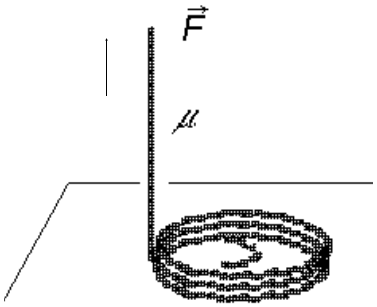
The extra mass adds ma^2 to the moment of inertia. Thus the new sweet spot is located at

$$s_{new} = \frac{I_{tot}}{(M+m)\,x_{new}} = \frac{I_{yy} + Mx^2 + ma^2}{Mx - ma} = s + 2\ \text{cm}\ .$$

This is easily solved for m to get

$$m = M\,\frac{x \cdot 2\ \text{cm}}{a\ (s + a + 2\ \text{cm})} \approx 0.27\ \text{kg}\ .$$

2. A uniform rope of mass μ per unit length lies coiled on a table. An upward force F is applied to one end of the rope. Find the maximum height $y_{\max}$ the rope's end reaches, and the time T it takes to get there. Describe qualitatively the motion after the maximum height is reached.



Solution

We use Newton's 2nd Law in the form $F = \frac{dp}{dt}$:

$$F - \mu y g = \frac{d}{dt}\left(\mu y \dot{y}\right)$$

where y is the current height of the end of the rope. Multiplying by the integrating factor $y\dot{y}$ (this should be obvious by now!) we find

$$\frac{F}{\mu} y\dot{y} - g y^2 \dot{y} = y\dot{y}\,\frac{d}{dt}\left(y\dot{y}\right)$$

i.e.

$$\frac{F}{\mu}\frac{d}{dt}\left(\frac{y^2}{2}\right) - g\frac{d}{dt}\left(\frac{y^3}{3}\right) = \frac{1}{2}\frac{d}{dt}\left(y\dot{y}\right)^2$$

or

$$\frac{F}{\mu}\left(\frac{y^2}{2}\right) - g\left(\frac{y^3}{3}\right) = \frac{1}{2}\left(y\dot{y}\right)^2 + \text{constant}\,.$$

Since $y(0) = \dot{y}(0) = 0$, the constant also is 0, hence we can divide out a factor of y^2 to get

$$\frac{F}{\mu} - \frac{2gy}{3} = \dot{y}^2\,.$$

Defining

$$y_{\max} = \frac{3F}{2g\mu}$$ (this is obviously the maximum height!)

the latter equation becomes

$$\dot{y} = \pm\left(\frac{2}{3} g\,(y_{\max} - y)\right)^{1/2}$$

which, upon integration, yields

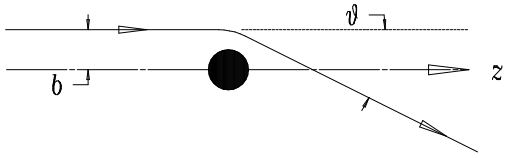
$$T \equiv \int_0^T dt = \left(\frac{3}{2g}\right)^{1/2} \int_0^{y_{\max}} dy\,\left(y_{\max} - y\right)^{-1/2} = \left(\frac{6y_{\max}}{g}\right)^{1/2} = \left(\frac{9F}{\mu g^2}\right)^{1/2}.$$

After reaching $y_{\max}$, the acceleration becomes $\ddot{y} = -\frac{1}{3}g$, so the rope starts back down. In fact, it is easy to see that the acceleration is constant, so the rope continues downward until it is once more on the table. Then it starts upward again. The falling time is also T, so the period is $2T$.

3. A particle of large kinetic energy E scatters from the potential

$$V(r) = \frac{k}{r^2 + R^2}$$

where R is a constant. Calculate the scattering angle as a function of impact parameter in the *small angle approximation*.



Solution

The momentum gain perpendicular to the beam direction (in this case the y-direction) is

$$\Delta p_y = \int_{-\infty}^{+\infty} dt \left(-\frac{\partial V(r)}{\partial y} \right).$$

The angular deflection is then

$$\Delta\theta \approx \frac{\Delta p_y}{p_z} \approx \frac{1}{m v_z^2} \int_{-\infty}^{+\infty} dz \left(-\frac{\partial V(r)}{\partial y} \right) = \frac{ky}{E} \int_{-\infty}^{+\infty} \frac{dz}{\left(z^2 + y^2 + R^2 \right)^2} = \frac{\pi}{2} \frac{V(y)}{E} \frac{y}{\sqrt{y^2 + R^2}}.$$