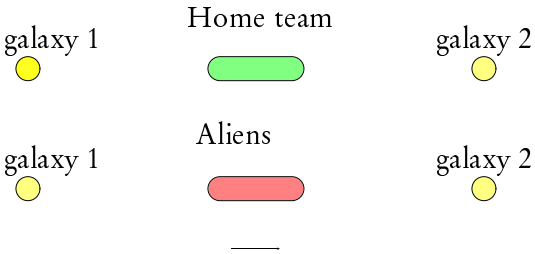
# Solutions to HW #1

Ohanian, p. 122

1. According to the observer in the lab, the events took place at space-time positions

$$\begin{pmatrix} t_1 \\ x_1 \end{pmatrix} = \begin{pmatrix} -L/c \\ -L \end{pmatrix}, \quad \begin{pmatrix} t_2 \\ x_2 \end{pmatrix} = \begin{pmatrix} -L/c \\ L \end{pmatrix}.$$




The′ moving observer sees the events at

$$\begin{pmatrix} t'_1 \\ x'_1 \end{pmatrix} = \gamma \begin{pmatrix} -L/c + vL/c^2 \\ -L + vL/c \end{pmatrix}$$

$$\begin{pmatrix} t'_2 \\ x'_2 \end{pmatrix} = \gamma \begin{pmatrix} -L/c - vL/c^2 \\ L + vL/c \end{pmatrix}$$

Now the time light takes to travel from galaxy 1 to the spaceship is

$$\tau_1 = \frac{1}{c} x'_1 + \frac{v}{c} \tau_1 = \frac{\gamma L}{c}$$

and that from galaxy 2 to the spaceship is

$$\tau_2 = \frac{1}{c} x'_1 - \frac{v}{c} \tau_2 = \frac{\gamma L}{c}$$

*i.e.*, the same. Thus the light from galaxies 1 and 2 arrive at ship times

$$t'_1 + \tau_1 = \gamma \frac{vL}{c^2}, \quad t'_2 + \tau_2 = -\gamma \frac{vL}{c^2}.$$

Since $L/c = 10^7$ yrs and $v/c = 10^{-4}$, $\gamma \approx 1$ and so the light from galaxy 2 arrives 2000 yrs before that from galaxy 1.

2. Clearly, the speed is $v = L/t$ so the difference in speed is

$$\Delta v = \frac{L}{t_0 - \Delta t/2} - \frac{L}{t_0 + \Delta t/2} \approx \frac{L \Delta t}{t_0^2}.$$

Since $t_0 \geq L/c$, the *maximum* difference in speed is

$$\frac{1}{c} \Delta v \approx \frac{c \Delta t}{L} = \frac{12 \text{ sec}}{1.6 \times 10^5 \text{ yrs}} = 2.4 \times 10^{-12}.$$

Assuming "a few" means ±3 hours, we see by similar reasoning that the maximum difference in speed between neutrinos and photons is

$$\frac{1}{c} \Delta v \approx \frac{c \Delta t}{L} = \frac{6 \text{ hrs}}{1.6 \times 10^5 \text{ yrs}} = 4.3 \times 10^{-9}.$$

5. Since proper (ship's) time and lab time are related by
$t = \gamma\tau$,
and since the distance is $L = vt$, we have
$v\gamma\tau = L$, $\frac{v}{c}\gamma = \frac{L}{c\tau} = \frac{46}{20}$, $\frac{v}{c} = \frac{1}{\sqrt{1+(20/46)^2}} = 0.917$.

The time on Earth would appear to be 50.16 yrs.

7. As we saw in class, if the invariant interval is spacelike, the order of two events can be reversed by Lorentz transformation (*i.e.* two observers in relatively moving frames can see the events in reversed order). In the stationary (w.r.t. Earth) frame, the events occur at
$\begin{pmatrix} t_1 \\ x_1 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$ and $\begin{pmatrix} t_2 \\ x_2 \end{pmatrix} = \begin{pmatrix} 0.0003\text{ s} \\ 160\text{ km} \end{pmatrix}$. The interval is

$s^2 = 160^2 - (3\times10^5\text{ km/s} \times 0.0003\text{ s})^2 = 17{,}500\text{ km}^2 > 0$.
Hence one can find a frame in which the first happens later than the second: we want
$t'_1 - t'_2 = \gamma\left(t_1 - t_2 - \frac{v}{c}(x_1 - x_2)\right) = \gamma\left(-0.0003\text{ s} + \frac{v}{c}\cdot 160\text{ km}\right) > 0$,
hence
$\frac{v}{c} > \frac{0.0003 \times c}{160} = \frac{9}{16}$ and $\gamma > \frac{16}{5\sqrt{7}} = 1.21$.