HW #2

1. The space-time points A, B have a timelike separation. Show that the integral

$$\int_A^B d\tau \equiv \int_A^B \sqrt{dx^\mu \, dx_\mu}$$

is maximum when the world-line $x^\mu(\tau)$ between the points is a straight line.

2. A collection of free particles has 4-momenta $p(i)$, $i=1, \ldots, N$ in the lab. Show that the square of their total 4-momentum,

$$P^\mu P_\mu = \left(\sum_{i=1}^{N} p^\mu(i) \right) \left(\sum_{i=1}^{N} p_\mu(i) \right)$$

is a Lorentz-invariant quantity, equal to the square of the total energy in a frame where the total 3-momentum vanishes. Find the Lorentz transformation connecting this "barycentric" frame with the lab frame.

3. Explain why a photon cannot spontaneously turn into an electron-positron pair ($m_e c^2 = 0.511$ MeV).

4. A photon collides with a heavy nucleus of mass M and produces an $e^+ e^-$ pair ($m_e << M$). What is the minimum ("threshold") energy the photon must have to make this process occur?

5. A collision between a proton with kinetic energy T and a proton that is stationary in the laboratory frame produces the charmed baryon-antibaryon pair $\Lambda_c \, \overline{\Lambda}_c$ in the reaction

$$p + p \rightarrow p + p + \Lambda_c + \overline{\Lambda}_c \, .$$

The mass of the proton is 938 MeV, and that of the Λ_c (and $\overline{\Lambda}_c$) is 2281 MeV. What is the minimum (laboratory) kinetic energy T needed to produce the $\Lambda_c \, \overline{\Lambda}_c$ pair?