HW #4

Ohanian, Chapter 2, problems 26 and 28.

Extra problem:

1. The relativistic Lagrangian of a free particle is

$$L = -mc^2\sqrt{1-\vec{u}\cdot\vec{u}/c^2}\,.$$

a) Use this to find an expression for the 3-momentum of the particle *via* $\vec{p} = \dfrac{\partial L}{\partial \vec{u}}$.

b) The Hamiltonian (energy) is defined as $H = \vec{u}\cdot\vec{p} - L$. Hence find an explicit expression for the total energy.

c) Do the above expressions for momentum and energy agree with what we found before?

2. Suppose the above particle finds itself in external central potentials: $V(r)$ that resembles the time component of a 4-vector; and $S(r)$ that behaves like a scalar quantity (it combines with the mass); so that the Lagrangian becomes

$$L = -\Big(mc^2 + S(r)\Big)\sqrt{1-\vec{u}\cdot\vec{u}/c^2} - V(r)\,.$$

What is the resulting 3-momentum, energy and equation of motion? From Newton's Law identify the force on the particle.