HW #5

1. Show that the electromagnetic dual tensor,

$$\widetilde{F}^{\mu\nu} \overset{df}{=} \varepsilon^{\mu\nu\alpha\beta}\left(\partial_\alpha A_\beta - \partial_\beta A_\alpha\right)$$

automatically satisfies

$$\partial_\mu \widetilde{F}^{\mu\nu} = 0 \qquad \text{(second pair of Maxwell's equations).}$$

2. Show that the electromagnetic dual tensor $\widetilde{F}^{\mu\nu}$ and the electromagnetic tensor $F^{\mu\nu}$ have opposite behavior under reflection of the space coordinates or time-reversal, $t \to -t$.

(**Hint**: evaluate $\widetilde{F}^{\mu\nu} F_{\mu\nu}$ in terms of $\vec{E}$ and $\vec{B}$ and use the 3-dimensional form of Maxwell's equations to establish the effects on these fields of space- or time reflections.

3. The {00} component of the energy-momentum tensor is the Hamiltonian (energy) density in a field theory described by the Lagrangian density $\mathscr{L}(\varphi, \partial_\mu \varphi)$ is

$$\mathscr{H} = \partial_t \varphi \, \frac{\partial \mathscr{L}}{\partial(\partial_t \varphi)} - \mathscr{L} ;$$

a) What is the (obvious) generalization for $T^{\mu\nu}$?

(**Hint:** you must symmetrize the expression explicitly.)

b) Use the equations of motion (Euler-Lagrange equation) to show that $T^{\mu\nu}$ is conserved.

4. Show that adding a "mass" term $\delta\mathscr{L} = \frac{1}{8\pi}\frac{m^2 c^2}{\hbar^2} A^\mu A_\mu$ to the electromagnetic Lagrangian density $\mathscr{L} = \frac{-1}{16\pi} F^{\mu\nu} F_{\mu\nu}$ leads to the "Coulomb" potential from a point charge at the origin

$$A^0(r) = \frac{Q}{r} e^{-\kappa r} ,$$

and express the "cutoff distance" $\frac{1}{\kappa}$ in terms of the constants found in $\delta\mathscr{L}$.