HW #8

1. A white dwarf star has the solar mass and a radius ≈ 9000 Km . What is the average Fermi momentum of the electrons (in MeV/c), assuming the star is made of fully ionized helium? Are the electrons relativistic or nonrelativistic?

2. Estimate the tidal stress acting on a rod of cross-section A and length L falling lengthwise through the event-horizon of a black hole of mass $M = 2\times10^{33}$ gm . Assume the rod is steel: what is the Roche's limit for the rod (smallest distance it can approach the black hole)?

3. Show that the trajectory of light rays in the Schwarzschild metric obeys

$$\frac{d^2u}{d\varphi^2} + u = 3u^2$$

where $u = \dfrac{MG}{r}$, r is the radial coordinate and φ is the azimuthal angle.

4. Paul is orbiting a neutron star at a distance 4MG/c , in a circular orbit. Peter, his twin brother, is fired radially outward from the surface with less than escape velocity. His path crosses Paul's orbit just as Paul comes by, so they synchronize their clocks. On his way back down, Peter again encounters Paul as their trajectories cross, Paul having completed 10 orbits between their meetings. They again compare clocks. How much do their clocks disagree?

5. A two-dimensional surface can be represented in 3-dimensional Cartesian coordinates as

$$\left(\frac{x}{a}\right)^2 + \left(\frac{y}{b}\right)^2 + \left(\frac{z}{c}\right)^2 = 1\,,$$

where a, b and c are constants.

a) Find a suitable two-dimensional (parametric) representation of the surface.

b) Find an expression for the (differential) distance between two points *in the surface*, thereby constructing the metric tensor.

c) Find the *shortest* distance, in the surface, between the points $z = +c$ and $x = +a$.

d) Construct the curvature tensor, the Ricci tensor and the curvature scalar.

e) What is the area of the surface? (Express it as an integral, it can't be done in closed form.) Check limiting cases to verify your result.

Hints: Examine the spherical case to see what to do about part a). In part b) the answer is *not* $(dx)^2+(dy)^2+(dz)^2$. Use vectorial methods to restrict displacements to the surface.