HW # 10

1. Consider a 3-dimensional space of constant curvature, whose metric is given by

$$(ds)^2 = d\vec{r} \cdot d\vec{r} + \frac{(d\vec{r} \cdot \vec{r})^2}{a^2 - \vec{r} \cdot \vec{r}}$$

a) What is the curvature of the space?

b) What is the shape of the space for $a^2 > 0$ as well as $a^2 < 0$?

c) Assuming $a^2 > 0$, what is the radius and the circumference of the circle of constant $r = b$, whose center is the origin, $r = 0$?

Hint: Use polar coordinates,

$$x = r \sin\theta \cos\varphi$$
$$y = r \sin\theta \sin\varphi$$
$$z = r \cos\theta \ .$$

2. Define 2 vectors in Minkowski space

$$A_\mu = \left(0, -1, 0, 0\right)$$

and

$$B_\mu = \left(0, 0, -1, 0\right).$$

From these, define the two polarization tensors

$$\varepsilon^{(1)}_{\mu\nu} = A_\mu B_\nu + A_\nu B_\mu$$

and

$$\varepsilon^{(2)}_{\mu\nu} = A_\mu A_\nu - B_\nu B_\mu \ .$$

Show that under rotations through angle φ about the z-axis the two tensors transform as

$$\varepsilon^{(1)}_{\mu\nu} \rightarrow \varepsilon^{(1)}_{\mu\nu} \cos(2\varphi) - \varepsilon^{(2)}_{\mu\nu} \sin(2\varphi)$$

$$\varepsilon^{(2)}_{\mu\nu} \rightarrow \varepsilon^{(1)}_{\mu\nu} \sin(2\varphi) + \varepsilon^{(2)}_{\mu\nu} \cos(2\varphi)$$

What do you deduce from this result, about the spin of the gravitational quantum?

3. Solve the Oppenheimer-Volkov equation 25.16 (using the expressions 25.17a,b for energy density and pressure) numerically, in the same manner as you solved the Chandrasekhar equation. Find the maximum mass.

NOTE: I misled you—the Chandrasekhar equation does not become unstable w.r.t collapse. The reason for collapse to a neutron star is the inverse β-decay process that causes loss of electron pressure.