Gravitation and Cosmology, I

Newton's laws of mechanics and gravitation explain the motions of the planets and their satellites, as well as the motions of stars within galaxies. Astrophysics and nuclear physics provide a good understanding of the structure and evolution of stars. Cosmology is the study of the origin, evolution and structure of the entire universe. Its ultimate aim is to explain the large-scale features (structure larger than galaxies) that we observe through optical and radio telescopes, cosmic-ray detectors and satellite-borne gamma- and x-ray detectors.

Cosmology attempts to answer such questions as

- "What is the age and size of the universe?";
- "Why do the chemical elements have their observed relative abundances?";
- "Since matter and antimatter seem to be completely symmetric, why do we not observe antimatter in the extrasolar particles (cosmic rays) continuously bombarding us?"

As the result of major observational discoveries made in the past two decades,

- We now believe the universe had a definite beginning, $10–20 \times 10^9$ years ago, in an explosion called the "big bang".
- We believe the present abundance ratios of the elements (particularly hydrogen, deuterium and helium) reflect the conditions at the beginning of the universe.
- We believe we can now explain the absence of antimatter (more precisely, the presence of matter) in terms of the physical laws governing particle interactions at extremely high energies.

Two main findings of observational astronomy give insight into the origin of the universe. They are the systematic red-shift of light from distant galaxies ("cosmological red-shift") and the 3 ^{o}K black-body radiation permeating the universe.

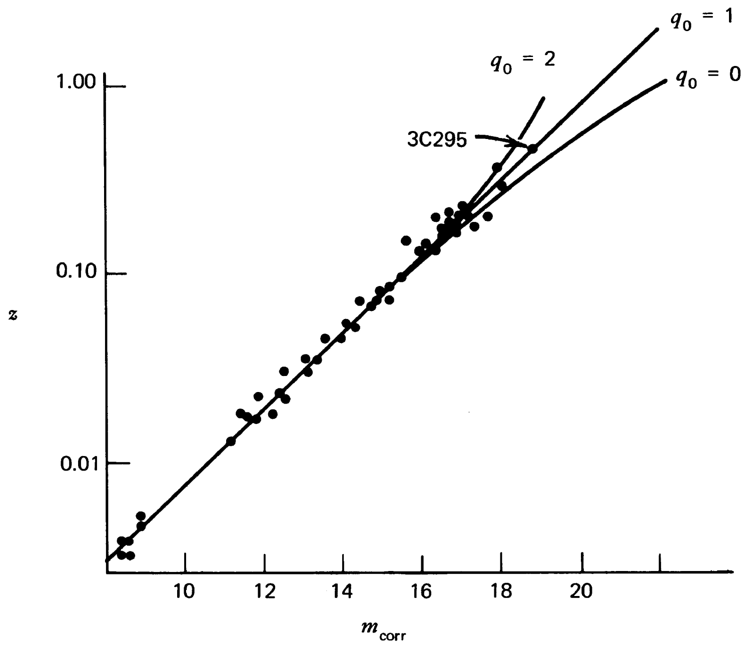
These phenomena are today believed to be the remnants of the colossal explosion which marked the beginning of the universe.

Cosmological Red shift

The frequencies of known spectral lines in the light from distant galaxies are typically shifted downward by a relative amount proportional to distance:

$$\frac{\Delta\nu}{\nu} = \frac{-HR}{c}. \qquad (14.1)$$

In Eq. 14.1 H is "Hubble's constant", R is the distance from the Earth and c is the speed of light. This effect is called the cosmological red shift. The cosmological red shift is apparently independent of the direction of observation. We reproduce below a plot of red shift *vs.* (corrected) apparent magnitude†, for 42 first-ranked cluster galaxies.



Example

A bright galaxy is determined to be 4×10^9 light-years from our own galaxy. What is the ratio of the frequency of a spectral line in the light from this galaxy, to that from a reference light source in the laboratory? Take H to be 15 km/sec/10^6 *l-y*.

Solution

The relative frequency shift, from Eq. 14.1 is

$$\Delta\nu/\nu = -4\times10^9\ l\text{–}y \times 15\ \text{km/s}/10^6\ l\text{–}y\ /\ 3\times10^5\ \text{km/s}$$

$$= 0.20\ .$$

Thus $\nu'/\nu = 1 - 0.20 = 0.8$

If we interpret the frequency shift Eq. 14.1 as a Doppler shift, its independence of direction means that the distant galaxies are receding from a common center---the Solar system---at velocities proportional to their distance from the center

$$v = HR\ . \tag{14.2}$$

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† S. Weinberg, *Gravitation and Cosmology* (John Wiley and Sons, Inc., New York, 1972), p. 448.

Equation 14.2 is what we would expect to see if a tremendous, spherically symmetric explosion had taken place, with us at its precise center, and with all the other matter expelled outward from the center.

At any time after the explosion, the most distant matter would be that initially given the largest outward velocity. The distance of any particular piece, at time would be $R = vt_0$. If the explosion picture is the explanation of Eq. 14.2, then we must interpret the Hubble constant H as $1/t_0$, where t_0 is the time elapsed since the explosion.

Example
The Hubble constant is determined by observation to be roughly

$$H = 15 \text{ km/sec/ } 10^6 \text{ light–yr} .$$

What is the time since the "explosion"?

Solution

The time t is just $1/H$, which is

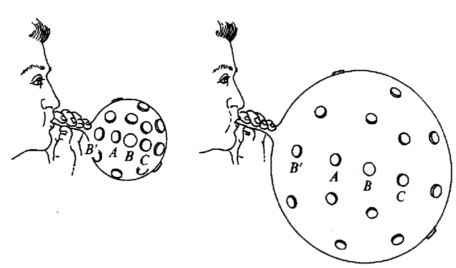
$$\frac{(10^6 \text{ lt–yr}) \times (3{\times}10^8 \text{ m/s}) \times (3{\times}10^7 \text{ s/yr})}{1.5{\times}10^4 \text{ m/s}} = 6{\times}10^{17} \text{ sec} = 2{\times}10^{10} \text{ yr} .$$

The center

People once believed that the Earth was the center of the universe. Does the uniform recession of the distant galaxies mean they were correct, that the Earth (or anyway, our own Milky Way galaxy) is the actual center of the universe?

Today we believe the laws of nature permit no definite center. Translation invariance (as reflected in momentum conservation) makes clear that there is no specific preferred origin of coordinates.

This would imply an observer in another galaxy would see the distant matter receding from him, as though his galaxy were the center of things. How is it possible for both ourselves and distant observers to see uniformly receding galaxies, as though both were simultaneously at the center of creation? We can explain this seeming paradox with the two-dimensional analogy of a spherical balloon, upon whose surface someone has randomly placed dots, as in the figure to the right.



The surface of the balloon is supposed to be a two-dimensional representation of our three-dimensional space (the 3-dimensional "surface" $t = const.$ in 4-dimensional space-time). All "distances" between pairs of dots are the shortest curves connecting those dots in the surface of the balloon. Now inflate the balloon.

As the figure† makes clear, each dot gets farther from each other dot, by an amount proportional to their initial separation. Let one of the dots stand for our galaxy: then we would observe (remember, light has to travel on the shortest path lying in the surface of the balloon) all the other dots receding uniformly from us at a velocity

$$v = \frac{R}{K}\frac{dK}{dt}\,. \qquad (14.3)$$

In Eq. 14.3 K is the radius of the sphere, and $R(t)$ is the present separation of two dots (as measured in the surface). In this picture we identify the Hubble constant H with the relative expansion rate of the sphere,

$$H = \frac{1}{K}\frac{dK}{dt} \equiv \frac{d\log(K)}{dt}\,. \qquad (14.4)$$

Because the balloon is spherical, any observer (on any dot) sees the same thing as any other. Thus no dot can be identified as the "absolute center" of the surface.

Example

Since light cannot be redshifted to negative frequencies, what is the effective size of the universe?

Solution

Taking the Hubble constant to be

$$H = 15\,\text{km/sec}/10^6\,\text{lt–yr}\,,$$

we set the largest possible recession velocity equal to c, to get

$$c = HR\,.$$

Thus

$$R_{Universe} = \frac{c}{H} = \frac{3\times10^5\ \text{km/s}}{(15\ \text{km/s/M–ly})} = 2\times10^{10}\ \text{l–y}\,.$$

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† after Misner, Thorne and Wheeler, *Gravitation*.