Gravitation and Cosmology, II

In the previous lecture we saw that the the red-shift of light from distant objects follows a distinct pattern: the apparent velocity of recession obeys Hubble's Law,

$$v = HR ,$$

where R is the distance and H the Hubble constant. In other words, the universe seems to be expanding, pointing to an explosive origin.

The helium/hydrogen ratio.

A difficult theoretical problem in cosmology is to explain the relative abundances of hydrogen, deuterium and helium.

In the "steady state" cosmological model, the universe has always been present (that is, it has no beginning or end).

The cosmological expansion indicated by the red shift (which means the average matter density would decrease with time) can be reconciled with a steady state universe only if new matter is continously formed to "fill in the gaps".

Because hydrogen is fused into heavier elements in stars and supernovas, the steady state model leads to equilibrium ratios of helium to hydrogen abundance. In fact, there would be very little helium, because any helium formed in a star eventually gets burned up in forming ^{12}C and heavier elements.

But the observed ^{4}He/H ratio is about 25%. The Big Bang model explains this large ratio: if the universe were initially at temperatures of billions of degrees, the blackbody radiation would be so energetic and intense it would dissociate heavy elements as fast as they were formed. As the universe expanded and its temperature cooled to less than 10^9 $^{\mathrm{o}}$K, the rate of the forward reaction

$$n + p \rightarrow d + \gamma \tag{14.1}$$

would begin to exceed the rate of the reverse photodissociation reaction

$$\gamma + d \rightarrow n + p . \tag{14.2}$$

Now it would become possible to build up ^{4}He through proton and neutron capture reactions on deuterium. Because the universe continued to expand and cool conditions were no longer appropriate for helium burning into heavier elements. Thus the Big Bang itself produced almost no elements heavier than helium[†].

[†] We now believe all the elements up to iron were formed in thermonuclear reactions in red giant stars; whereas all the elements above iron in the Periodic Table were formed by the intense neutron bombardment accompanying the many Type II supernova explosions that have taken place since the formation of stars.

Detailed calculations of the Big Bang explosion agree with observed relative abundances of ^{1}H, ^{2}H and ^{4}He, if the density of electromagnetic radiation was very high[†]. (In fact, the ratio of the number of photons to the number of baryons had to have been of order 10^{9} to 10^{10}.)

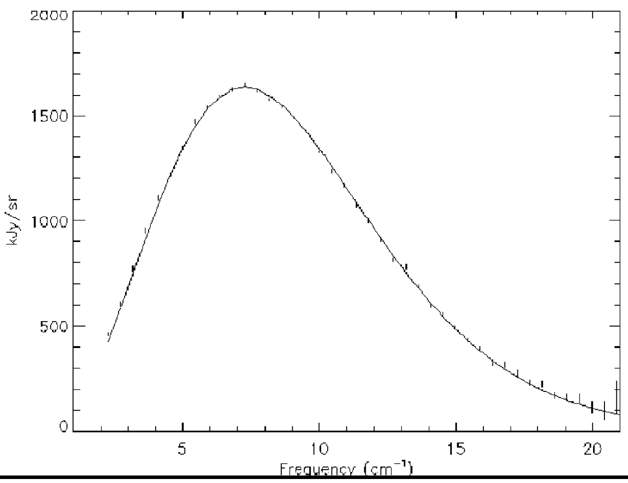
Cosmic microwave radiation background

The second piece of observational evidence for the explosive origin of the universe is the 3° K cosmic microwave radiation background (1965, Penzias and Wilson, Nobel Prize in physics, 1983). Why is this radiation important, and what does it tell us?

If the Big Bang explanation of the cosmological ^{4}He/H ratio is correct, then in the early universe electromagnetic radiation had the frequency spectrum of a blackbody at extremely high temperature. The average photon had more than enough energy to ionize atoms or create $e^{+}e^{-}$ pairs.

At this early stage in the history of the universe, matter would have been opaque to radiation, so that the particles and radiation remained in thermodynamic equilibrium. When the temperature fell below a few thousand degrees, however, pair production and photoionization became energetically impossible. Suddenly matter would have been transparent to electromagnetic radiation.

As the universe continued to expand and cool, the evolution of radiation and of matter followed different paths: protons and ^{4}He nuclei captured electrons to make hydrogen and helium gas, but the decoupled radiation retained its blackbody spectral distribution. The cosmological expansion of all distances gradually increased the wavelengths in the cosmic blackbody radiation, until today the peak frequency has a characteristic temperature of only 3° K. Photons of this low energy (≈1/4000 eV) have frequencies in the microwave region. The Figure below shows data from the COBE satellite with the Planck blackbody spectrum for $T = 2.7$ °K .



The cosmic microwave radiation background can be detected with extremely sensitive, low-noise microwave antennas and receivers. In fact, it was detected accidentally by Penzias and Wilson, who were trying to eliminate a persistant source of noise in a new type of microwave antenna they had constructed at Bell Laboratories in New Jersey[‡]. Professors Dicke and Peebles of

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[†] The most straighforward discussion of this can be found in Bernstein, Brown and Feinberg, *Rev. Mod. Phys.* **61** (1989) 25. See also Ohanian and Ruffini, Chapter 10.

[‡] They even investigated the possibility that a “white dielectric substance” left behind by nesting birds, was the noise source.

Princeton University suggested Penzias and Wilson had observed the radiation left over from the Big Bang. Measurements at various microwave frequencies have shown that the spectral distribution of the cosmic microwave background agrees with Planck's blackbody radiation formula, with $T=2.7\ ^{o}K$.

Moreover, the radiation is isotropic----its intensity and frequency distribution are independent of direction[†].

"Big bang" cosmology

A rocket fired into space has initial kinetic energy

$$KE = \frac{1}{2} mu^2 \tag{14.3}$$

at a point near the Earth's surface. Its initial (gravitational) potential energy is

$$V = \frac{-mMG}{R} \tag{14.4}$$

where M is the Earth's mass, R its radius, and G the universal gravitational constant.

The rocket will either escape from the Earth or fall back to the surface, depending on whether or not KE exceeds $|V|$.

Now imagine a sphere of matter of radius R large compared with a galaxy, but small compared with the size of the universe, and a galaxy at the surface of this sphere, moving outward with speed $u = HR$. Exactly as with the rocket, if

$$\frac{H^2 R^2}{2} > \frac{GM}{R} \tag{14.5}$$

the galaxy will escape the gravitational attraction of the sphere of matter; whereas if the inequality is reversed, the galaxy will eventually fall back. The mass of the matter is just

$$M = \frac{4\pi}{3} R^3 \bar{\rho} ,$$

where $\bar{\rho}$ is the average density of matter in the universe. Inserting this expression for M into Eq. 14.5 we find

$$H^2 R^2 > \frac{4\pi}{3} GR^2 \bar{\rho} . \tag{14.6}$$

Clearly R^2 cancels from both sides of Eq. 14.6 so the destiny of the galaxy----whether it will escape or whether it will fall back----is determined by the average density of matter. If

$$\bar{\rho} > \frac{3H^2}{8\pi G} , \tag{14.7}$$

[†] Barring effects of the Earth's motion relative to the barycentric frame of the cosmic black body radiation, that is.

the gravitational force will eventually slow and reverse the direction of the receding galaxy.

Conversely, if

$$\bar{\rho} < \frac{3H^2}{8\pi G}, \qquad (14.8)$$

the galaxy will escape. The quantity

$$\rho_{crit} \overset{df}{=} \frac{3H^2}{8\pi G} \qquad (14.9)$$

is called the *Einstein critical density*----the density of mass-energy just sufficient to slow the rate of expansion to zero after infinite time has passed.

Example
What is the critical density predicted by the observed Hubble constant H?

Solution

The Hubble constant is

$$H = 15\ \text{km/sec} / 10^6\ \text{lt-yr} = 1.7 \times 10^{-18}\ \text{sec}$$

giving

$$\rho_{crit} = \frac{3}{8 \times 3.1416} \cdot (1.7 \times 10^{-18})^2 \cdot \frac{1}{6.67 \times 10^{-8}} = 5 \times 10^{-30}\ \text{gm/cm}^3.$$

Astronomers can estimate the density of matter in the universe by adding up the masses of all the matter they observe. This includes stars (whose masses are are related to their luminosities and spectra by the theory of stellar structure); clouds of gas and dust (some of which occludes starlight and some of which glows by fluorescence from microwave to visible wavelengths); and various forms of non-luminous (dark) matter (that reveals itself through the gravitational binding of star clusters).

When all this matter is accounted for, the average density of the universe is between 2 and 10 times smaller than the critical density.

Many scientists think the observed density is too close to ρ_{crit} to be a coincidence. Because numerical coincidences are philosophically unsatisfactory, they would prefer a theory in which the actual density has to exactly equal the critical density .

Is the universe open or closed? We could answer this question if we knew the deceleration parameter q_0, which measures the rate gravitation slows the initial expansion of the universe.

The deceleration parameter is approximately defined by

$$R(t) = R_0 \left[1 + Ht - \frac{1}{2} q_0 (Ht)^2 \right], \qquad (14.10)$$

where t is measured from the present. To derive Eq. 14.10 simply note that, taking the present as $t = 0$, we have

$$R(t) = R_0 + v_0 t + \frac{1}{2} a_0 t^2 \tag{14.11}$$

where v_0 is the present velocity, $R_0 H$, and a is the present acceleration, $-R_0 H^2 q_0$ (This defines q_0 as a dimensionless ratio.) According to General Relativity, the universe is open if q_0 is 0.5, closed if it exceeds 0.5.

Present measurements are very crude, but give values for q_0 in the neighborhood of 0.5 . That is, the deceleration also suggests that the observed density of matter in the universe is close to critical. At the moment, we do not know—from observational evidence—whether the universe is open or closed.

Relation to particle physics

High energy physics has greatly increased our understanding of gravitation (General Relativity), and the behavior of matter under rather bizarre circumstances of extreme pressure, temperature and density.

We can now construct realistic mathematical models that trace the evolution of the universe after the Big Bang†.

The details of the Big Bang itself—together with certain difficulties in the "standard model" of the evolution of the universe—are concealed in the very first, extraordinarily tiny fraction of a second ($t < 10^{-20}$ sec). Until quite recently no theory existed that permitted further backward extrapolation in time.

Grand unified field theories (GUTs) of elementary particles have suggested interesting answers to these difficulties, as well as suggesting a reason for the Big Bang itself.

Matter vs. antimatter

The symmetry between matter and antimatter in relativistic quantum mechanics implies that a symmetric universe—that started out with a bang—should contain equal amounts of each. Locally—Earth, Sun, Solar System—only matter is found. Where is the antimatter? GUTs tell us we have been viewing the problem from the wrong end. We have regarded the amount of matter (baryons) in the universe as very large. This is wrong—in terms of particle count, the universe contains mostly photons. In fact, the measured ratio of photons to baryons (from the blackbody background radiation) is astronomically large: 109 or 10^{10}. GUTs explain this ratio as follows:

> GUTs predict the existence of (hitherto unobserved) very massive particles, X and $\overline{X}$, that eventually decay into baryons and antibaryons. In the first 10^{-35} seconds or so after the Big

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† S. Weinberg, *The First Three Minutes.*

Bang, the temperature was large enough to produce local thermodynamic equilibrium of all particles, including X and $\overline{X}$. At this point there would have been precisely equal numbers of particles and antiparticles. As the universe cooled, the $X\overline{X}$ pairs would rapidly decay.

But in GUTs there is a slight asymmetry[†] in the decays of the X's and $\overline{X}$'s . If baryon number is not conserved (as a finite lifetime for the proton would indicate), the asymmetric decays of X and $\overline{X}$ would create a very slight imbalance between the numbers of protons and antiprotons—perhaps 1 part in 10^9 or 10^{10} difference.

As matter cooled and photons fell out of equilibrium with pp pairs, more pairs would annihilate than were produced. Eventually, all the available pairs annihilated, leaving only those protons which had no matching antiprotons.

Just enough electrons would have been produced during these asymmetric decays to balance the charge of the left-over protons. In other words, all the matter existing today results from this incredibly tiny asymmetry between particles and antiparticles.

Missing mass problem

We see apparently gravitationally bound systems in the universe (globular clusters, clusters of galaxies). Presumably there is enough mass to bind them. Estimates of the mass yield ratios

$$\frac{\mathit{observed\ mass}}{\mathit{needed\ mass}} \approx 0.1 \text{ to } 0.3, \tag{14.12}$$

depending on the size of the cosmological objects being observed. Where is the missing mass? In the GUTs there are 6 kinds of neutrino (including antineutrinos), all of which can have masses. Possibly the mass is hiding in the form of low-energy neutrinos emitted both in the Big Bang and subsequently during the radioactive processes of stellar evolution . Although the electron neutrino's mass is measured[‡] to be <12 eV , the muon neutrino mass could be as large as 0.5 MeV, and the tau neutrino mass is currently known to be less than ≈ 164 MeV (although no one believes either ν_μ or ν_τ is that massive). So stopped, massive neutrinos are a possibility.

[†] This asymmetry is observed experimentally in the decays of K^0 and $\overline{K}^0$ mesons.

[‡] The best present bound comes from the end-point of the ^{3}H β-decay. The observation of neutrinos within ±2 hrs of the light from the supernova SN1987A sets a less stringent limit: $m_\nu < 1000$ eV.

The Big Bang

Why did the Big Bang take place, and why is the actual density of matter close to the Einstein critical density, Eq. 14.9?

Some theories liken the early universe to a superheated fluid. The temperature is extremely high, but the pressure is too low to prevent density fluctuations. The system can lower its free energy by boiling, that is, going from a high energy but symmetric state, to a lower energy asymmetric state.

If the system is unstable any little fluctuation can grow, leading to a transition from the uniform to the boiling state. The energy released in this transition now becomes available to inflate the bubbles. Our present universe is the interior of one such bubble.

This scenario, called a “grand inflationary universe”, requires an almost perfect balance between mass-energy and gravitational energy. That is, it requires the total energy----mass-energy plus gravitation----to be zero, to an extraordinarily accurate approximation.

The inflationary universe model thus answers two questions at once: in the beginning the universe formed out of “nothing” because “nothing” is unstable; and the observed mass is close to the critical mass because our patch of “nothing” has zero total energy.