PH524 Midterm Take-Home Examination

Do all 5 problems. Due: 4 March '98 @ 12:00 (noon).

1. A Cepheid variable star is 100 Kpsec away from the Earth, and a large unknown mass is directly in the line of sight, at an unknown distance from Earth. The gravitational lens effect (predicted by Einstein) says such a source will appear as a ring centered on the (invisible) occluding mass.

a) What is the angle subtended by the ring, as a function of the (unknown) mass of the intervening object and its (unknown) distance from Earth?

b) Suppose the angular size of the ring is 3" of arc. Express the unknown mass in solar masses, its unknown distance as a fraction of the Cepheid variable's distance from Earth, and find a relation between the (unknown) mass and its (unknown) distance.

(Hint: use the Einstein formula for deflection of light by a gravitational field)

2. A photon of energy 5 eV collides head-on with an electron of energy 20 GeV (1 GeV $= 10^9$ eV) and scatters elastically. What is the maximum energy of the scattered photon? At what angle (relative to its initial direction) is it scattered?

(Hint: use conservation of 4-momentum.)

3. In class we derived the Lienard-Weichart potential

$$A^{\mu}(x) = Q \frac{d\xi^{\mu}}{dt'} \left| |\vec{x} - \vec{\xi}(t')| - \frac{d\vec{\xi}}{dt'} \cdot \left(\vec{x} - \vec{\xi}(t')\right) \right|^{-1} \qquad (9.11)$$

for the vector potential of a moving charge, where

$$t' = t - |\vec{x} - \vec{\xi}(t')|$$

is the "retarded time". Use this expression to calculate the Poynting vector

$$\vec{S} = \frac{1}{4\pi} \vec{E} \times \vec{B}$$

far from an electric charge that is stopping in matter with constant deceleration, whose position is

$$\vec{\xi}(t) = \left(u\,t - \frac{1}{2} a t^2\right) \hat{z}.$$

Calculate the total rate at which electromagnetic energy is radiated as a function of time.

(Hint: Use the relation $|\vec{x} - \vec{\xi}| \approx |\vec{x}| - \hat{x} \cdot \vec{\xi}$, $|\vec{x}| >> |\vec{\xi}|$, and don't forget that $\frac{d\xi^{\mu}}{dt'}$ depends on both position and time.)

4. Neutrinos (average energy 15 MeV) from the 1987A supernova were detected, within a time-frame ±3 hours, in coincidence with the light from the explosion. Since the supernova is about 150,000 light-years away, this coincidence permits us to set an upper bound on the *mass* of the electron neutrino. Estimate this upper bound.

   Also note that the photons and neutrinos passed through the *same* (spatially varying) gravitational potential supplied by the intervening matter. Estimate the maximum (gravitational) time-delay that would be caused by a mass equal to that of our galaxy (take its mass to be $10^{11}\, M_{\odot}$ and its radius to be 30,000 l-y), What does the fact that light and neutrinos arrived almost at the same time imply about neutrinos?

5. A massive scalar field theory is defined by the Lagrangian density

$$\mathscr{L} = \frac{1}{2}\left(\partial_\mu \varphi\ \partial^\mu \varphi\ -\ m^2\, \varphi^2\ \right) - \frac{\lambda}{4}\left(\varphi^2\ -\ f^2\ \right)^2$$

   where m and f are real constants.

   a) What is the Hamiltonian density of the field?

   b) What is the total 3-momentum carried by the field? (Not the momentum-density!)