Due: Tuesday, 5 May 1998 @ 17:00.

1. A 2-dimensional surface embedded in Euclidean 3-space is parameterized by

$$x = \zeta \cos\varphi$$
$$y = \zeta \sin\varphi$$
$$z = \zeta$$

a) What are the elements of the metric tensor $g_{\alpha\beta}$? (let $x^1 = \zeta$, $x^2 = \varphi$)

b) Calculate the scalar curvature R. Is the surface open or closed?

c) Why should the answer to b) above be unsurprising?

2. Some theorists have suggested that a neutron star can undergo (weak) interactions that transform neutrons to Λ hyperons *via* the reactions

$$n \longleftrightarrow \Lambda + \pi^0$$
$$\Lambda \longleftrightarrow n + \pi^0$$
$$p \longleftrightarrow \Lambda + \pi^+$$
$$\Lambda \longleftrightarrow p + \pi^- \ .$$

Useful information:

$$M_\Lambda = 1115.6\,\mathrm{MeV/c^2}$$
$$M_n = 939.6\ \mathrm{MeV/c^2}$$
$$M_p = 938.3\ \mathrm{MeV/c^2} = 1.67\times10^{-24}\ \mathrm{gm}$$
$$m_{\pi^0} = 135.0\ \mathrm{MeV/c^2}$$
$$m_{\pi^\pm} = 139.6\ \mathrm{MeV/c^2}$$
$$M_\odot = 1.99\times10^{33}\ \mathrm{gm}$$

Assume practically all the neutrons in a neutron star have been converted to Λ's. What is the relative concentration of neutrons and protons to that of Λ's? Take

$$M_{star} = M_\odot$$
$$R = 10\ \mathrm{Km}$$

(**Hint:** What would be the momentum of the emitted nucleon in the decay of Λ?)

3. Consider the polarization tensors for a gravitational wave propagating in the z direction:

$$\varepsilon^{(1)}_{\mu\nu} = A_\mu B_\nu + A_\nu B_\mu$$

$$\varepsilon^{(2)}_{\mu\nu} = A_\mu A_\nu - B_\mu B_\nu$$

where

$$A_\mu = \left(0, -1, 0, 0\right)$$

$$B_\mu = \left(0, 0, -1, 0\right) .$$

Find linear combinations $\hat{t}_{\pm}$ of $\varepsilon^{(1)}_{\mu\nu}$ and $\varepsilon^{(2)}_{\mu\nu}$ that, under rotations about the z-axis by an angle θ, transform as

$$\hat{t}_{\pm}' = e^{\pm 2i\theta}\,\hat{t}_{\pm} \ .$$

4. A binary star consists of a neutron star and a black hole, respectively of masses $1.3\,M_{\odot}$ and $5\,M_{\odot}$. Their orbital separation is 100 Km, and the orbit is circular. What is the rate of decrease of their orbital separation, due to the loss of energy in gravitational radiation?

5. What is the "proper" distance—as we defined it in class and notes—of a quasar with an apparent recession velocity $0.95c$? Take the Hubble constant to have the value

$$H_0 = 15\times 10^{-6}\ \text{km/sec/light–year}\ ,$$

and assume the universe to be asymptotically flat with cosmological constant 0.

6. According to a "BELIEVE IT OR NOT!" cartoon (hyperlinked to this page)
"The amount of energy Americans use every day chewing bubble gum could light a city of 10 million inhabitants."

a) What is wrong with the way this problem is phrased?

b) Make a reasonable correction to the phrasing and determine whether the statement is true.

c) Estimate the radiated power (in gravitational waves) from all this gum-chewing.

d) How does this compare with the electromagnetic radiation that would be emitted if the point of each gum-chewer's chin had one (1) unit of electron charge attached to it?