Phys 531 Assignment 5 Due October 3

1. In general, a lossless beam splitter can be characterized by its complex transmittance t and complex reflectance r. In other words, when a wave U_{in} is incident on the beam splitter, a wave rU_{in} is reflected from the front surface and a wave tU_{in} is transmitted out the rear surface. Conservation of energy evidently requires that $|r|^2 + |t|^2 = 1$.

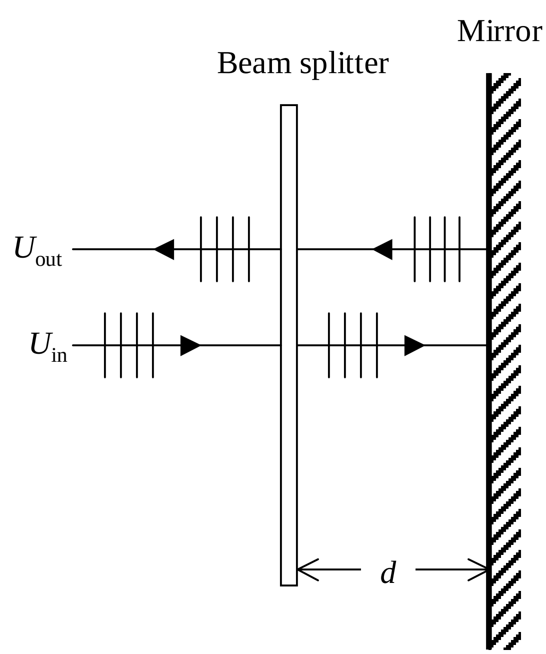
A less obvious relation between r and t can be obtained by considering the peculiar interferometer shown below, in which the beam splitter is placed a distance d in front of a perfectly reflecting plane mirror. Suppose that light acquires a phase ϕ_M when it bounces off the mirror.

(a) By summing the multiply reflected fields, find an expression for the total wave reflected by the interferometer.

(b) By energy conservation, the intensity of the reflected wave must equal the intensity of the incident wave, regardless of the spacing d. Using this fact, show that if $r = |r|\exp(i\beta)$, then t must be

$$t = |t|e^{i(\beta \pm \frac{\pi}{2})}$$

Hint: if an equation of the form $A + Be^{i\phi} + Ce^{-i\phi} = 0$ holds for all ϕ, then A, B and C must all be zero.



2. Suppose a laser produces pulses of light 50 fs in duration, repeated at a 100 MHz rate (1 fs $= 10^{-15}$ s). The central wavelength of the light is 800 nm.
(a) How many optical cycles occur during one pulse?
(b) Approximately what range of wavelengths of light is present?
(c) Averaged over long times ($\sim$1 s), the laser's power output is 0.1 W. Estimate the peak optical power during a single pulse.

3. Compute the Fourier transforms F_ν of the following functions, and sketch both $f(t)$ and $|F_\nu|^2$:

(a) $$f(t) = \begin{cases} 0 & \text{if } t < 0 \\ \sin(2\pi\nu_0 t) & \text{if } 0 \le t \le T \\ 0 & \text{if } t > T \end{cases}$$

Take $\nu_0 T = N$ for integer N.

(b) $$f(t) = \exp\left(-\frac{|t|}{T}\right)$$

4. Consider the sum of two monochromatic waves in free space:

$$u(\mathbf{r}, t) = A\cos(2\pi\nu_1 t - k_1 z) + A\cos(2\pi\nu_2 t - k_2 z)$$

with $\nu_1 \approx \nu_2$.
(a) Find the complex representation $U(\mathbf{r}, t)$ for this wave.
(b) Calculate the intensity $I(\mathbf{r}, t)$. At what speed does the interference pattern move in space?