Phys 531 Assignment 6 Due October 10

1. Show that the paraxial wave $U(\mathbf{r}) = A(\mathbf{r})\exp(-ikz)$ with amplitude

$$A(\mathbf{r}) = \frac{A}{z + iz_0}\exp\left[-ik\frac{x^2+y^2}{2(z+iz_0)}\right]$$

satisfies the paraxial wave equation. Calculate and sketch the intensity in the $z = 0$ plane. This wave is called a Gaussian beam, and it provides a reasonable model for a laser beam. (See also Exercise 2.2-2, pg 51.)

2. Saleh and Teich problem 4.1-1, page 153, parts (a)–(d), with the following modifications: Express your answers for arbitrary d (i.e., don't take $d = 10^4\lambda$), and don't bother to "describe the physical nature of the wave." (Hint: You don't need to do any integrals to solve these problems.)

3. Saleh and Teich problem 4.1-2, page 153.

4. (4 points) If the function

$$f(x, y) = A'\exp\left[-\frac{x^2+y^2}{W_0^2}\right]$$

represents the complex amplitude of an optical wave $U(x, y, z)$ in the plane $z = 0$, show that $U(x, y, z)$ is the Gaussian beam of problem 1, with $W_0^2 = \lambda z_0/\pi$:
(a) using the frequency-domain transfer function method
(b) using the space-domain impulse response method.
You can take $W_0 \gg \lambda$ and use the Fresnel approximation in either case. You'll probably want to review the calculation of the Fourier transform of a Gaussian function that we did in class. (See also Saleh and Teich exercise 4.1-2, page 121.)

5. We've discussed optical propagation in terms of transfer functions and impulse response functions. These methods can be applied to any linear problem. For instance: consider a driven damped harmonic oscillator, described by

$$\frac{d^2g}{dt^2} + 2\pi\sigma\frac{dg}{dt} + (2\pi\nu_0)^2 g = f(t)$$

Find the transfer function $\mathcal{H}(\nu)$, defined by $g(t) = \mathcal{H}(\nu)f(t)$ when $f(t)$ is a monochromatic function of frequency ν.
Extra Credit: Find the impulse reponse function for this problem, defined either as the response $h(t) = g(t)$ to a unit impulse $f(t) = \delta(t)$, or equivalently as

$$h(t) = \int_{-\infty}^{\infty} e^{2\pi i\nu t}\mathcal{H}(\nu)d\nu.$$

I'll keep track of extra credit points through the semester, and add them into your grade after the final curve is determined.