Phys 531 Assignment 9 Due October 31

1. The average intensity of sunlight at the earth's surface is 250 W/m^2. Consider a monochromatic plane wave in vacuum with this intensity and wavelength 550 nm. Calculate the amplitudes of
(a) the electric field (E_0)
(b) the magnetic flux density (B_0), and
(c) the time-averaged energy density.
Your answers should have the correct SI units.

2. Saleh and Teich, problem 5.3-1, page 191.

3. In a medium with an electric current density $\mathcal{J}$, Maxwell's equation (5.2-4) is modified to $\nabla \times \mathcal{H} = \mathcal{J} + \epsilon \partial \mathcal{E}/\partial t$, with the other equations unaltered. If the medium is described by Ohm's law, $\mathcal{J} = \sigma \mathcal{E}$, where σ is the conductivity, show that the Helmholtz equation, (5.3-15) is applicable with a complex valued k. Give an explicit expression for k in terms of ω, ϵ, μ_0, and σ. (See also Saleh and Teich, problem 5.5-1, page 192.)

4. We discussed in class that the index of refraction n represents a shorthand method for taking into account the motion of charges bound within a dielectric medium. It is alternatively possible to include the bound charges explicitly:
(a) Use Maxwell's equations to calculate the density ρ of bound charge in terms of the polarization density of the medium $\mathcal{P}(\mathbf{r})$. (Assume no free charges are present.)
(b) In an inhomogeneous linear medium, $\mathcal{P} = \epsilon_0 \chi(\mathbf{r}) \mathcal{E}$, where we allow the susceptability χ to vary in space. Calculate ρ in terms of χ and the applied $\mathcal{E}$. You may find useful the identity

$$\nabla \cdot (\psi \mathbf{F}) = \mathbf{F} \cdot \nabla \psi + \psi \nabla \cdot \mathbf{F},$$

which holds for any scalar function ψ and vector function $\mathbf{F}$.
(c) Consider specifically a uniform plane wave normally incident on one face of a homogenous glass cube. The electric field of the light is aligned with the edges of the cube. Describe the spatial distribution of the bound charges in this case.
(d) From these results, you should conclude that in a infinite homogeneous medium, the bound charge density is everywhere zero. Yet wave propagation is affected, since the index of refraction is not 1. If this effect is not due to ρ, what term in Maxwell's equations does it come from?